QCD and its Success

1. Lagrangian (non-abelian gauge theory)

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i \sum_q \bar{\psi}^i_q \gamma^\mu (D_\mu)_{ij} \psi^j_q - \sum_q m_q \bar{\psi}^i_q \psi_{q_i}$$

$$F^{(a)}_{\mu\nu} = \underbrace{\partial_\mu A^a_\nu}_{\substack{\text{gluon} \\ (a=1,2,\cdots 8)}} - \partial_\nu A^a_\mu - \underbrace{g_s}_{\substack{\text{Strong} \\ \text{Coupling} \\ \text{constant}}} \underbrace{f_{abc}}_{\substack{\text{structure} \\ \text{constant} \\ \text{of} \\ SU(3)}} A^b_\mu A^c_\nu, \quad (a, b, c = 1, 2, \cdots 8)$$

(Note: In QED, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, which is an abelian gauge theory.)

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu + i g_s \sum_a \underbrace{\frac{\lambda^a_{ij}}{2}}_{SU(3)\text{ generators}} A^a_\mu, \quad (i, j = 1, 2, 3)$$

m_q: mass of quarks.

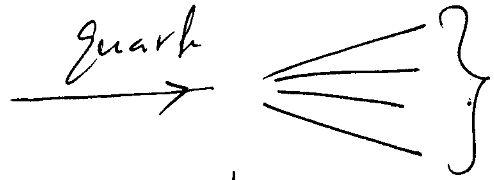
(1) non-perturbative QCD:

① We don't see free quarks.

② Quarks have to fragment into hadrons.

$\Rightarrow$ (Parton-hadron duality)

(Field-Feynman fragmentation model)



③ The relevant energy scale μ is

$\mu \simeq \Lambda_{QCD}$ (where α_s becomes larger than 1)

$\Lambda_{QCD} \simeq 200$ MeV

(2) Perturbative QCD

① We only deal with quarks and gluons, not hadrons.

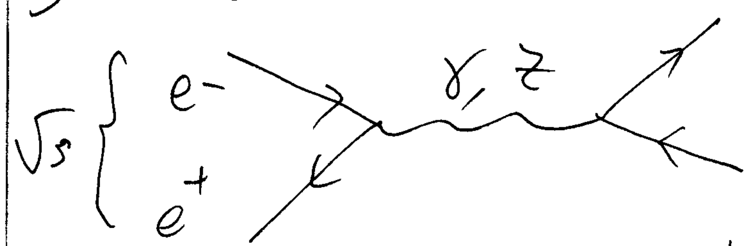
② The relevant scale μ is

$\mu > \Lambda_{QCD}$

(Typically $\mu > 10$ GeV, pQCD is reliable.)

2. Perturbative QCD in $e^- e^+$ collisions

1) $\sigma(e^- e^+ \to \text{hadrons})$ for $\sqrt{s} < M_Z$



too heavy for this method

where $q = u, d, s, c, b, \cancel{t}$ (depending on $\sqrt{s} > 2m_q$)

Ignoring the mass of final state fermions, then the factor, for low values of $\sqrt{s}$,

$$R \equiv \frac{\sigma(e^- e^+ \to \text{hadrons})}{\sigma(e^- e^+ \to \mu^- \mu^+)}$$

$$= 3 \sum_q e_q^2 \qquad \left(\text{for } m_q < \frac{\sqrt{s}}{2} \text{ and } \sqrt{s} < M_Z \right)$$

Beyond tree level,

$$R = R^{(0)} \left[1 + \frac{\alpha_s}{\pi} + C_2 \left(\frac{\alpha_s}{\pi} \right)^2 + C_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \cdots \right]$$

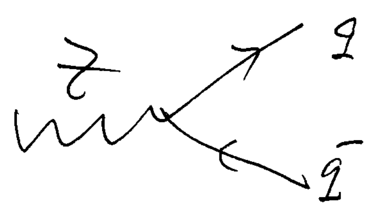
where

$$C_2 = 1.411 \qquad \alpha_s \equiv \frac{g_s^2}{4\pi}$$

$$C_3 = -12.8$$

2) At Z-pole, $\sqrt{s} \simeq M_Z = 91$ GeV

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \mu^+\mu^-)}$$



$$= 20.767 \pm 0.025 \Rightarrow \alpha_s(M_Z) \simeq 0.12$$

(1) Partial decay widths

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi}$$

CKM matrix element

$$\Gamma(W^+ \to u_i \bar{d}_j) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \cdot C \cdot |V_{ij}|^2$$

where $C = 3\left[1 + \frac{\alpha_s}{\pi} + C_2\left(\frac{\alpha_s}{\pi}\right)^2 + C_3\left(\frac{\alpha_s}{\pi}\right)^3\right]$

(Quark has colors.) (Same as in R.)

$$\Gamma(Z \to \psi_i \bar{\psi}_i) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}\left[(g_V^i)^2 + (g_A^i)^2\right] \cdot C_i$$

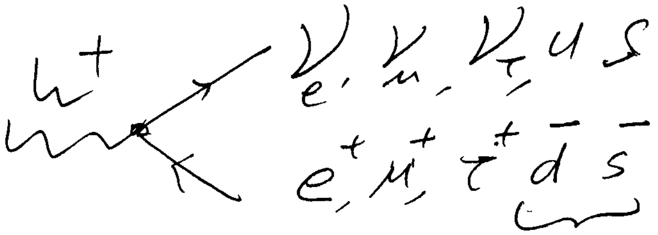
where $C_i = 1$ for leptons (ν, e, ...)

$C_i = C$ for quarks

$$\frac{-g_W}{2\cos\theta_W}\gamma^\mu(g_V^i - g_A^i\gamma_5)$$

(2) Decay branching ratios

$Br(W^+ \to e^+ \nu_e) \simeq \frac{1}{9}$



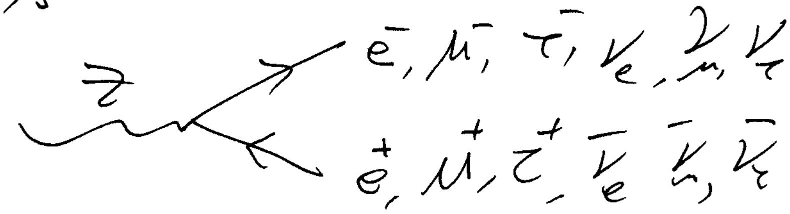
$\left(1+1+1+3+3 = 9 \right)$
channels
with same coupling strength

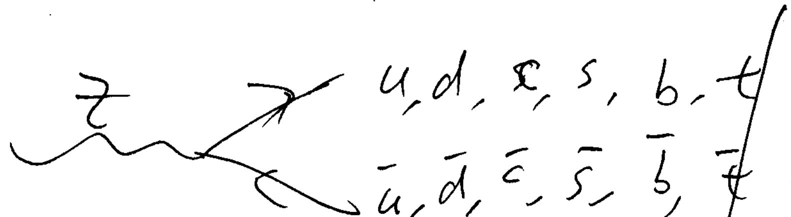
$Br(Z \to e^+e^-) \simeq 3.4\%$

$Br(Z \to u\bar{u}) \simeq 11.6\%$

$Br(Z \to d\bar{d}) \simeq 15.6\%$

$Br(Z \to e^+e^-) \equiv \frac{\Gamma(Z \to e^+e^-)}{\Gamma(Z \to all)}$



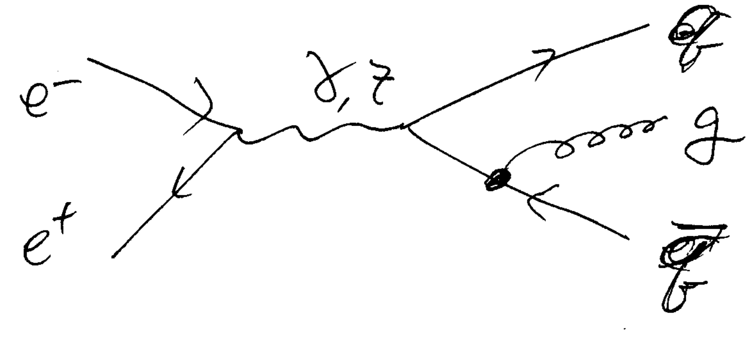


too heavy

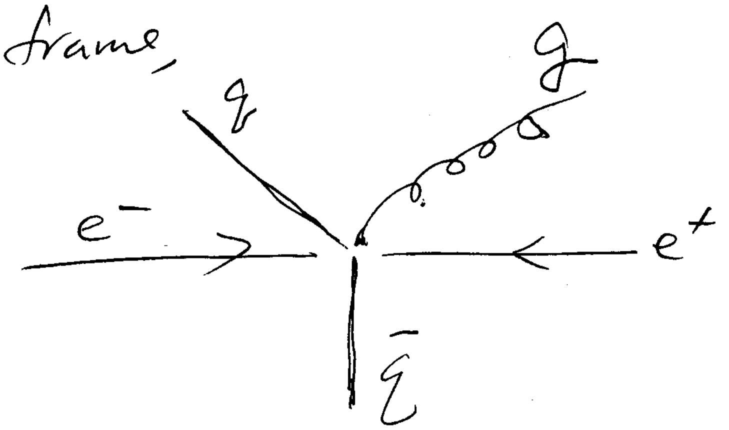
$\boxed{\begin{array}{l} M_W = 80.4\ \text{GeV} \\ \Gamma_W = 2.1\ \text{GeV} \end{array}}$

$\boxed{\begin{array}{l} M_Z = 91.2\ \text{GeV} \\ \Gamma_Z = 2.5\ \text{GeV} \end{array}}$

3) Gluon was found in e^-e^+ Collisions (1976)



Or, in e^-e^+ c.m. frame,



The spin and color factor of gluon were verified.

3. Deep Inelastic Scatterings (DIS) in Lepton-Hadron Collisions

4. Hadron Collider physics