Standard Model of Cosmology

1. The simplest global description of the Universe. It's based on

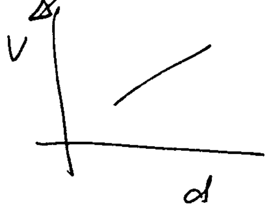
— General Relativity, (Einstein 1915)

— Cosmological Principle
(The three-spaces of constant time are homogneous and isotropic, at all times.)

— "Standard model" of particle physics (plus candidates for dark matter and large enough CP-violation) to describe the particle content of the early Universe.

2, Important historical events.

— 1929, Hubble estimated the distance to 18 galaxies, and concluded "roughly linear relation" between velocities and distances. (from Doppler shifts)



⇒ Confirm Cosmological principle

(1) (homogeneous and isotropic)

(2) Hubble Constant (recent measurement)

$$H \sim 15 \frac{Km}{s\cdot(Mly)}$$ million light years

(1 Mpc ≃ 3.26 Mly), (pc = Parsec)

(3) "Age" of universe < ("characteristic expansion time" $\simeq 1/H$ $\simeq 20 \times 10^9$ years)

— 1930s & 1940s

$H \sim 170 \frac{km}{s \cdot (Mly)}$

$\Rightarrow$ "Age" of universe < 2×10^9 years

But, from the relative abundance of various radioactive isotopes (e.g. U-235 and U-238) in the earth, the earth is about 4.6×10^9 years old.

— 1950s

Recalibration of the Cepheid period-luminosity relation (for Cepheid variables stars).

$\Rightarrow H \sim 15 \frac{km}{s \cdot (Mly)}$

$\Rightarrow$ solve "Age paradox"

$\Rightarrow$ Big-bang Cosmology becomes a standard theory. (1964)

(started ~~1950s~~ late 1940s)

3. Story of Λ (Cosmological Constant)

1) In 1917, Einstein tried to find a solution to his equations that would be homogeneous, isotropic, and, unfortunately, "static".
(To be consistent with the cosmological ideas at that time.)

⇒ He was forced to add a term, Λ, Cosmological Constant.

⇒ predicts no red-shifts.

2) In 1917, another solution of Einstein's modified theory was found by de Sitter.

⇒ His solution satisfies Cosmological Principle, and predicts red-shifts (proportional to distance).

3) In 1922, red-shift data was compiled in terms of the de Sitter model.

4) In 1929, Hubble discovered the proportionality of red shifts to distance.
(He knew about de Sitter's prediction.)

5) In 1922, Friedmann found the general homogeneous and isotropic solution to the original Einstein equations (without Λ).

$\Rightarrow$ This is the base of the most modern cosmological theory.

$\Rightarrow$ The galaxies are moving apart because they were thrown apart by some sort of explosion in the past.

$\Rightarrow$ The Universe is expanding, but decelerating.

6) Now (at 2000's), we observed the Universe is expanding, and accelerating.

$\Rightarrow$ Based on original Einstein theory, there is "Dark Energy" to be understood.

Or, based on modified Einstein theory (with Λ) we need to understand why Λ is so small (by 10^{-120}) as compared to Planck scale. $\left[\left(\frac{3\times10^{-4}\,eV}{10^{19}\,GeV}\right)^4 \simeq 10^{-126}\right]$

4. A few estimates about Universe

1) The critical density $\rho_c \sim H^2$

Proof: Consider a sphere of galaxies of radius R

① with density ρ. The mass of the sphere is

$$M = \frac{4\pi R^3}{3}\rho$$

② The total energy of any typical galaxy (with mass m) at the surface of the sphere is

$$E = P.E. + K.E.$$

with

$$P.E. = \frac{-mMG}{R}$$

Hubble law ($V = HR$)

$$K.E. = \frac{1}{2}mV^2 = \frac{1}{2}m(H^2R^2)$$

$$\Rightarrow \quad E = mR^2\left[\frac{1}{2}H^2 - \frac{4}{3}\pi\rho G\right]$$

It remain constant as the Universe expand.

$$\Rightarrow \quad \rho_c = \frac{3H^2}{8\pi G}$$

(Note: $R(t)$, $H(t)$, $\rho(t)$ vary as ~~time~~ t.)

Note: $1\,^\circ K = 8.6 \times 10^{-5}$ eV ($1\ kT = 8.6\times10^{-5}$ eV) k: Boltzmann Constant

$1\,^\circ K = 0.29$ cm $\left(\lambda_{max} = 0.2\frac{\hbar c}{kT}\right)$

2) The characteristic expansion time scale

$$t_{exp}(t) \equiv \frac{1}{H(t)} \sim \frac{1}{\sqrt{\rho(t)}}$$

Proof: ① The cosmic mass density $\rho(t)$ at time t is

$$\rho(t) \sim \begin{cases} T^4 \sim \left[\frac{1}{R(t)}\right]^4, & \text{for radiation-dominant era (because } u \sim T^4) \\ \left[\frac{1}{R(t)}\right]^3, & \text{for matter-dominant era } (\Rightarrow \text{the number of particles remain constant}) \end{cases}$$

Note: The total energy density in the black-body radiation at the temperature T is

$u \sim T^4$ (Stefan-Boltzmann law)

② Hence, from ①, as $R(t) \to 0$, $\rho(t) \to \frac{1}{R^n}$ (for $n=3$ or 4)

$\Rightarrow$ To keep E constant, we have, for

$R(t) \to 0$, $\quad \frac{1}{2} H^2(t) \to \frac{4}{3}\pi \rho(t) G$

$$E = mR^2(t)\left[\frac{1}{2}H^2(t) - \frac{4}{3}\pi\rho(t)G\right]$$

$$\Rightarrow t_{exp}(t) = \frac{1}{H(t)} = \sqrt{\frac{3}{8\pi\rho(t)G}}$$

Also, $H(t) \sim \frac{1}{\sqrt{\rho(t)}}$

3) The size of Universe at time t:

$$R(t) \sim \begin{cases} t^{1/2} & \text{for radiation-dominated era} \\ t^{2/3} & \text{for matter-dominated era} \end{cases}$$

Proof. Since

$$V(t) = \frac{dR(t)}{dt}$$

and

$$V(t) = H(t)\, R(t)$$

with

$$H(t) \sim \sqrt{\rho(t)} \sim \begin{cases} \left[\frac{1}{R(t)}\right]^2, & \text{for radiation} \\ \left[\frac{1}{R(t)}\right]^{3/2}, & \text{for matter} \end{cases}$$

$\Rightarrow$

$$R(t) \sim \begin{cases} t^{1/2}, & \text{for radiation} \\ t^{2/3}, & \text{for matter} \end{cases}$$

Note: (1) At any time t after the beginning, a "horizon" at a distance of order ct, from beyond which no information could yet have reached us.

(2) As $t \to 0$, $R(t) \sim t^{1/2}$, for radiation-dominant,

but the "horizon" $\sim t$.

$\Rightarrow$ At a sufficiently early time, any given "typical" particle is beyond the horizon.

4) The temperature at time t:

$$T(t) \sim \frac{1}{R(t)}$$

Note

① Freely expanding black-body radiation remains described by the planck formula, but with a temperature that drops in inverse proportional to the scale of expansion.

② The maximum of the planck energy distribution (for black-body radiation) occurs at

$$\lambda_{max} = 0.2 \frac{hc}{kT} \sim \frac{1}{T}$$

$$\Rightarrow \lambda_{max}(t) \sim R(t)$$

③ As long as thermal equilibrium is preserved, the total entropy of the Universe remain fixed.

$\Rightarrow SR^3 = \text{Constant}$, where S is the entropy per unit volume.

$\Rightarrow S \approx N_T T^3$ (for $R \sim \frac{1}{T}$)

where N_T is the effective number of species of particles whose threshold temperature lies below T.