Units in Gravity

1. Gravitational constant

$$[G_N] = (6.67\times10^{-11})\left(\frac{m^2}{kg}\right)\cdot\left(\frac{m}{s^2}\right) \qquad a = \frac{GM}{r^2}$$

$$\boxed{G_N = 6.71\times10^{-39}\ \hbar c\ \left(\frac{GeV}{c^2}\right)^{-2}}$$

(1) $\left.\begin{array}{l} m_e c^2 = 0.511\ MeV \\ m_e = 9.1\times10^{-31}\ kg \end{array}\right\} \Rightarrow 9.1\times10^{-31}\ kg\cdot c^2 = 0.511\ MeV$

$\Rightarrow kg \sim 0.5\times10^{30}\ MeV$

(2) $1\ s = 3\times10^{8}\ m = 3\times10^{8}\times10^{15}\ fm$

$\sim 3\times10^{23}\left(\frac{1}{197\ MeV}\right)$

$\sim 1.5\times10^{21}\left(\frac{1}{MeV}\right)$

(3) $1\ m \sim 10^{15}\ fm \sim 10^{15}\cdot\left(\frac{1}{197\ MeV}\right) \sim 0.5\times10^{13}\left(\frac{1}{MeV}\right)$

$$\Rightarrow [G_N] \sim \frac{m^3}{kg\cdot s^2} = \frac{(0.5\times10^{13})^3\cdot\frac{1}{(MeV)^3}}{0.5\times10^{30}\ MeV\cdot(1.5\times10^{21})^2\frac{1}{(MeV)^2}}$$

$$\sim 10^{-28}\ \frac{1}{(GeV)^2}$$

Define $\boxed{G_N \equiv \frac{1}{M_p^2}}$ $\qquad M_p$: planck mass

$\Rightarrow M_p \sim 10^{19}\ GeV$

[redacted]

Energy scale		Size $= \left[\frac{1}{\text{Mass}}\right]$
10^{18} GeV	M_P	10^{-34} m
	LHC	
1 TeV	Tevatron (weak scale ~ 246 GeV)	10^{-19} m
100 GeV	LEP II	10^{-18} m
200 MeV	(strong interaction)	10^{-15} m

$$\left(\begin{array}{l} xp \sim \hbar \\ \Rightarrow x \sim \frac{\hbar}{p} \\ \sim \frac{\hbar}{mc} \end{array} \right)$$

2. Compare the strength of interactions.
(Consider u & $\bar{u}$ quarks, with mass = 5 MeV)

(1) E.M. force

$$\frac{\alpha \left(\frac{2}{3}\right)^2}{r^2} \sim \frac{1}{137} \cdot \frac{1}{(1\,\text{fm})^2} \cdot \left(\frac{2}{3}\right)^2$$

(2) strong force

$$\frac{\alpha_s}{r^2} \sim \frac{1}{(1\,\text{fm})^2}$$

(3) Gravitational force

$$\frac{G_N m_u m_{\bar{u}}}{r^2} \sim \frac{6.7\times10^{-45} \left(\frac{1}{\text{MeV}}\right)^2 \cdot (5\,\text{MeV})(5\,\text{MeV})}{(1\,\text{fm})^2} \sim \frac{10^{-43}}{(1\,\text{fm})^2}$$

3. Combine Gravity with SM interactions (strong ⊕ weak ⊕ E.M)

$\Rightarrow$ Why is $M_p (\sim 10^{19}\,\text{GeV}) \gg v\ (\sim 246\,\text{GeV})$

$\Rightarrow$ Why is $\left(\frac{v^2}{M_p^2} \sim 10^{-33}\right)$ so small ?

4. Combine Gravity with Quantum Mechanics

1) Schwarzschild radius

$$r_s \equiv \frac{2 G_N M}{c^2}$$

$$r_s = \begin{cases} 2.95 \times 10^{3}\ \text{m} & \text{for Sun} \\ 8.86 \times 10^{-3}\ \text{m} & \text{for earth} \end{cases}$$

(If $r \leq r_s$, the metric coefficient g_{rr} becomes infinite. $\Rightarrow$ Space-time has no meaning.)

Any gravitational collapsed stars, and other objects smaller than their Schwarzschild radii, are called "black holes".

The collapse is very fast:

$$\Delta\tau \sim \frac{r_s}{c} \sim 10^{-5}\ \text{sec for a}$$

black hole, with mass of the order of Sun, lives after crossing r_s before being destroyed in the central singularity.

If we want to probe Planck scale (M_p)

then
$$r_s = \frac{2G_N M_p}{c^2} = \left(\frac{2G_N m_{sun}}{c^2}\right)\left(\frac{M_p}{m_{sun}}\right)$$
$$= (2.95\times10^{3}\,m)\left(\frac{2.17\times10^{-8}\,kg}{1.99\times10^{30}\,kg}\right)$$
$$\sim 3\times10^{-19}\,m.$$

But, from Q.M., the size of the M_p energy scale is $\frac{\hbar c}{M_p} \sim 10^{-34}\,m$.

$\Rightarrow$ Since $\left(\frac{\hbar c}{M_p}\right) \ll r_s$ (at M_p scale),

so there is contradiction between Gravity and Quantum Mechanics

$\Rightarrow$ $\boxed{\text{String Theory}}$ (there is no point-like particle. Particles are various excited modes of string.)

$\Rightarrow$ Extradimensions

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2 Space-time are not static parameters, they are dynamical.