FOUR PHONON ANHARMONICITY AND ANGULAR MOMENTUM EFFECTS IN EVEN-EVEN SPHERICAL NUCLEI.

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We introduce the annihilation and creation bose operators d_μ and d^+_μ of quadruple phonons ($\mu = 0, \pm 1, \pm 2$) and the coordinate $d^{(+)}_\mu$ and momentum $d^{(-)}_\mu$ combinations

$$d^{(\pm)}_\mu = d_\mu \pm (-1)^\mu d^+_{-\mu}, \tag{1}$$

The Hamiltonian is give by

$$H = H^{(2)} + H^{(4)} + H', \tag{2}$$

$$= \sum_\mu \left(d^+_\mu d_\mu + \frac{1}{2} \right) + \frac{\lambda}{4} \left[\sum_\mu (-1)^\mu d^{(+)}_\mu d_-\mu^{(+)} \right]^2 + H', \tag{3}$$

The term H' which contain all possible corrections to the potential & kinetic energies will be neglected.

The term $H^{(4)}$ can be rewritten as a sum over the intermediate angular momenta of the pair $L = 0, 2, 4$ as follows.

$$H^{(4)} = \sum_L \gamma_L ((d^{(+)} d^{(+)})_L (d^{(+)} d^{(+)})_L)_{00}, \tag{4}$$

here the parentheses indicates vector addition of the angular momenta. We can see that in (4) there are four coordinate quadruple $d^{(+)}$ for which there is only one state with total angular momentum $J = 0$. Therefore $H^{(4)}$ will depend only on one combination of the coefficients λ_L. Indeed by recoupling of the angular momenta, $H^{(4)}$ can be written as

$$H^{(4)} = \frac{1}{3} \sum_L \sqrt{2L+1} \lambda_L ((d^{(+)} d^{(+)})_L (d^{(+)} d^{(+)})_L)_{00}, \tag{5}$$

Where

$$\lambda_L = \sum_{L'=0,2,4} \frac{\lambda'}{\sqrt{2L'+1}} \left[\delta_{LL'} + 2(2L'+1) \left\{ \begin{array}{ccc} 2 & 2 & L \\ 2 & 2 & L' \end{array} \right\} \right], \tag{6}$$

The quantities in (6) satisfy the homogeneous equation

$$\lambda_L = \frac{1 + (-1)^L}{2} \sum_{L'} \frac{1 + (-1)^{L'}}{2} \left\{ \begin{array}{ccc} 2 & 2 & L \\ 2 & 2 & L' \end{array} \right\} \lambda_{L'} (2L'+1), \tag{7}$$

Plugging (7) back in (6) we get

$$\lambda_2 = \lambda_4 = \frac{2}{7}\lambda_0 = \frac{\lambda}{2}, \tag{8}$$

Since the potential energy term $H^{(4)}$ has a rotational symmetry in five dimensional space besides the regular rotation -invariant then the square of the five dimensional angular momentum is conserved and is given by the casimir operator,

$$C = \frac{1}{2}\sum_{\mu\mu'}\left[d^+_\mu d_{\mu'} - (-1)^{\mu+\mu'} d^+_{-\mu'} d_-\mu\right]\left[d_{\mu'} d_\mu - (-1)^{\mu+\mu'} d^+_{-\mu} d_{-\mu'}\right], \tag{9}$$

The eigenvalues of this operator is given by

$$C = \nu(\nu + k - 2) = \nu(\nu + 3), \tag{10}$$

in our case $k = 2l + 1 = 5$, ν is the number of bosons which do not enter into those pairs with total angular momentum $J = 0$. The total number of quanta N is given by

$$N = \sum_\mu d^+_\mu d_\mu = 2n + \nu, \tag{11}$$

Where n is the number of boson pairs with total angular momentum $J = 0$. We now introduce the annihilation and creation operators of $J = 0$ boson pairs

$$P = \frac{1}{2}\sum_\mu (-1)^\mu d_\mu d_{-\mu}, P^+ = \frac{1}{2}\sum_\mu (-1)^\mu d^+_\mu d^+_{-\mu}, \tag{12}$$

It is easily verified that

$$C = N(N + k - 2) - 4P^+P, \tag{13}$$

For definite N and ν we have

$$\langle \nu N/P^+P/\nu N\rangle = \frac{1}{4}(N - \nu)(N + \nu + k - 2), \tag{14}$$

We also introduce the operator P_0 such that $P_0 = \frac{1}{2}H^{(4)} = \frac{1}{2}N + \frac{1}{4}k$. The hamiltonian $H = H^{(2)} + H^{(4)}$ can be expressed in terms of these generators :

$$H = 2P_0 + \lambda(P + P^+ + 2P_0)^2, \tag{15}$$

In accordance with equation(14) the matrix elements of P in the basis $/\nu, N; JM\rangle$ are

$$\langle \nu, N - 2; JM/P/\nu, N; JM\rangle = \frac{1}{2}\sqrt{(N - \nu)(N + \nu + k - 2)}, \tag{16}$$

Energy Spectrum :

Now we introduce a canonical transformation which takes the operators P_0, P, P^+ into $\tilde{P}_0, \tilde{P}, \tilde{P}^+$

$$P_0 = \frac{1}{2}\left(\omega + \frac{1}{\omega}\right)\tilde{P}_0 - \frac{1}{4}\left(\omega - \frac{1}{\omega}\right)(\tilde{P} + \tilde{P}^+), \tag{17}$$

$$P = \frac{1}{4}\left(\sqrt{\omega} + \frac{1}{\sqrt{\omega}}\right)\tilde{P} + \frac{1}{4}\left(\sqrt{\omega} - \frac{1}{\sqrt{\omega}}\right)\tilde{P}^+ - \frac{1}{2}\left(\omega + \frac{1}{\omega}\right)\tilde{P}_0, \tag{18}$$

Substituting (17) & (18) back into (15) and imposing the condition of the cancellation of the terms of the type $(\tilde{P} + \tilde{P}^+)$ we get an equation which gives ω in terms of ν, k, and λ

$$\omega_\nu^3 - \omega_\nu = 4(\nu + \frac{k}{2} + 1)\lambda, \tag{19}$$

We are left with the hamiltonian

$$H = E(\nu, \tilde{n}) + \omega_\nu \bar{\lambda}_\nu \left[4(\tilde{n}\tilde{P} + \tilde{P}^+\tilde{n}) + \tilde{P}^2 + (\tilde{P}^+)^2\right], \tag{20}$$

The diagonalized part $E(\nu, \tilde{n})$ which depends on ν and $\tilde{n}$ is given by

$$E(\nu, \tilde{n}) = \frac{1}{4}\left(3\omega_\nu + \frac{1}{\omega_\nu}\right)\left(\nu + \frac{k}{2}\right) + 2\omega_\nu \tilde{n}\left\{1 + \bar{\lambda}_\nu\left[3(\tilde{n} - 1) + \nu + \frac{k}{2} + 1\right]\right\}, \tag{21}$$

Where we have introduced the renormalized coupling constant

$$\bar{\lambda}_\nu = \frac{\lambda}{\omega_\nu^3} = \frac{1 - \omega_\nu^{-2}}{4(\nu + \frac{k}{2} + 1)} \leq \bar{\lambda}_0, \tag{22}$$

from (19) we see that for not too small values of λ the asymptotic regime $\omega_\nu = f_\nu^{1/3}$ sets in very rapidly, where $f_\nu = 4(\nu + \frac{k}{2} + 1)\lambda$.

The energy of the ground state quasirotational band is given by(21) with $\tilde{n} = 0$

$$E(\nu, \tilde{0}) = \frac{1}{4}\left(3\omega_\nu + \frac{1}{\omega_\nu}\right)\left(\nu + \frac{5}{2}\right), \tag{23}$$

For $(\lambda = 0 \rightarrow \omega_\nu = 1)$ we have a purely equidistant spectrum

$$E(\nu, \tilde{0}) = \nu + \frac{5}{2}, \tag{24}$$

on the other hand for weak anharmonicity (*small* ν) we have

$$E(\nu, \tilde{0}) = \left[1 + \lambda\left(\nu + \frac{7}{2}\right)\right]\left(\nu + \frac{5}{2}\right), \tag{25}$$

For strong anharmonicity (*large* ν) we have

$$E(\nu,\tilde{0}) = \frac{3}{4^{2/3}}\lambda^{1/3}\left(\nu+\frac{7}{2}\right)^{1/3}\left(\nu+\frac{5}{2}\right) \tag{26}$$

The ratio of the energies of the excited states $E(\nu,\tilde{0})$ and the first exited state $E(2,\tilde{0})$ is given by:

$$R_\nu = \frac{E(\nu,\tilde{0}) - E(0,\tilde{0})}{E(2,\tilde{0}) - E(,\tilde{0})} \tag{27}$$

Let's denote every thing in terms of the total angular momentum J where $J = 2\nu$, then

$$E(\nu,\tilde{0}) = E\left(\frac{J}{2},\tilde{0}\right)$$

We will also use the following notation for energies,

$$E(J_s^+) = E\left(\frac{J}{2},\tilde{n}\right)$$

Where the subscript(s) is related to $\tilde{n}$, i.e for $\tilde{n}=0 \to s=1$, for $\tilde{n}=1 \to s=2$ and so on, so for example

$$\begin{aligned} E(0,\tilde{0}) &= E(0_1^+) \\ E(1,\tilde{0}) &= E(2_1^+) \\ E(1,\tilde{1}) &= E(2_2^+) \end{aligned}$$

And so on.

The ratio R_4 of the excitation energies of the levels 4_1^+ and 2_1^+ is

$$R_4 = \frac{E(4_1^+) - E(0_1^+)}{E(2_1^+) - E(0_1^+)} = \frac{9\left(3\omega_2+\omega_2^{-1}\right) - 5\left(3\omega_0+\omega_0^{-1}\right)}{7\left(3\omega_1+\omega_1^{-1}\right) - 5\left(3\omega_0+\omega_0^{-1}\right)} \tag{28}$$

and it varies from $R_4 = 2$ for a harmonic oscillator, to $R_4 = 2.09$ for strong anharmonicity.

The ground state energy with only one pair of coupled bosons "$J=0$" is

$$E(0_2^+) = \frac{5}{8}(3\omega_0+\omega_0^{-1}) + 2\omega_0(1+\frac{7}{2}\bar{\lambda}_0) \tag{29}$$

And the ratio of the excitation energies is:

$$R_0 = \frac{E(0_2^+) - E(0_1^+)}{E(2_1^+) - E(0_1^+)} = 2\left(1+\frac{7}{2}\bar{\lambda}_0\right)\frac{8\omega_0}{7\left(3\omega_1+\omega_1^{-1}\right) - 5\left(3\omega_0+\omega_0^{-1}\right)} \tag{30}$$

Which has the value of 2.553 for strong anharmonicity.

We see that the energy ratios R_ν given in (27) is parameter free, we try to describe the general trends by the introduction of a one-parameter interpolation:

$$R_J = R_J^{00} + \sigma J(J+1), \qquad J \geq 4. \tag{31}$$

$$\sigma = \frac{R_J - R_J^{00}}{J(J+1)} \tag{32}$$

Where R_J is the experimental value, and R_J^{00} is the value obtained from equation (27).

As we will see, this simple parameterization gives a reasonable description for soft spherical nuclei as well as for well deformed nuclei.

We also introduce the energy parameter w which is given by

$$w = \sigma(E_2 - E_0) \tag{33}$$

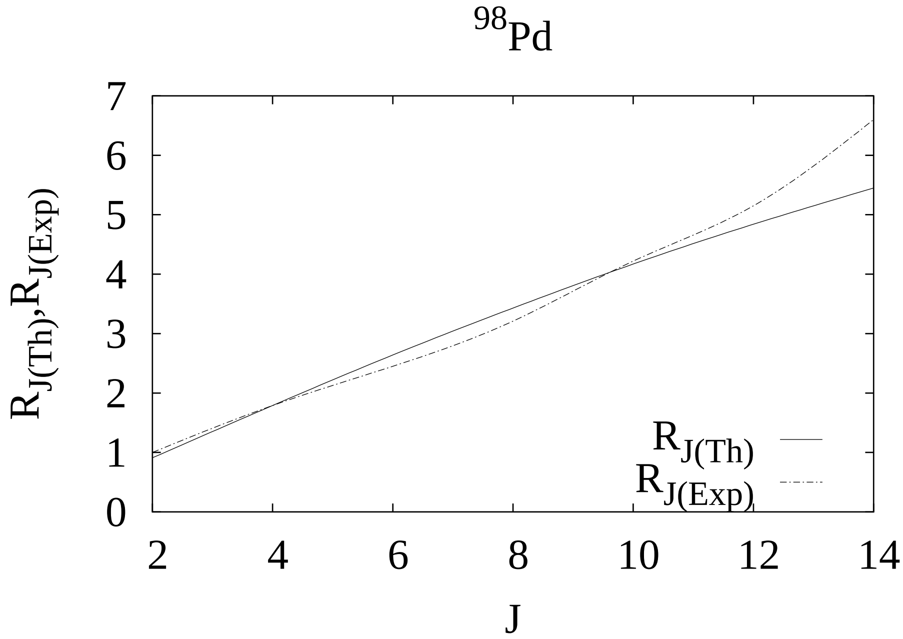
Analysis :

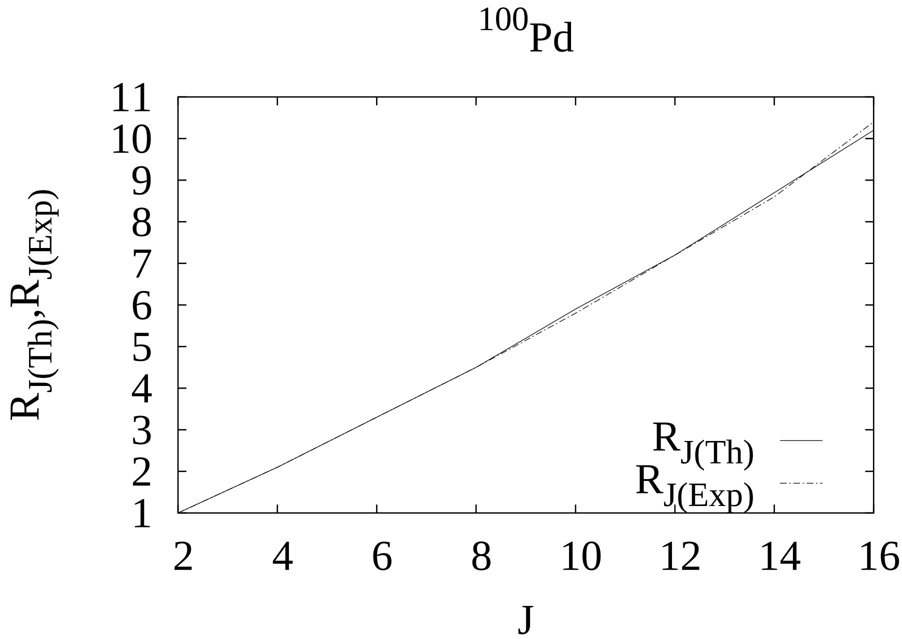
We studied the following isotopes :

^{100}Mo, ^{102}Mo, ^{104}Mo, ^{106}Mo, ^{108}Mo, ^{98}Pd, ^{100}Pd, ^{102}Pd, ^{104}Pd, ^{106}Pd, ^{108}Pd, ^{110}Pd, ^{112}Pd, ^{114}Pd, ^{116}Pd, ^{96}Ru,^{98}Ru, ^{100}Ru, ^{102}Ru, ^{104}Ru, ^{106}Ru, ^{108}Ru, ^{110}Ru ^{112}Ru, ^{98}Cd, ^{100}Cd, ^{102}Cd, ^{104}Cd ^{104}Cd, ^{106}Cd, ^{108}Cd, ^{110}Cd, ^{110}Cd ^{114}Cd, ^{116}Cd, ^{118}Cd, ^{76}Kr, ^{78}Kr, ^{80}Kr, ^{82}Kr, ^{84}Kr, ^{86}Kr, ^{76}Se, ^{78}Se, ^{80}Se, ^{82}Se, ^{84}Se.

For each isotope we calculated the energy $E(J)$ using equation (21),we also calculated the energy ratio $R_{J(Th)}$ which was done by first finding the R_J using equations (27) then using equation (32) we found σ for each isotope, then we took σ_{avg} for each isotope and multiplied it by $J(J+1)$ for each J, this gave us the second term in (31), then added this to R_J obtained originally form (27) to get $R_{J(Th)}$. We also calculated the parameter w using equation (33) . On a separate graph for each isotope we compared $R_{J(Th)}$ and $R_{J(Exp)}$ by plotting them versus J on the same plot. We also found the average value of σ for each isotope and then, for each element, we plotted the average values of σ versus the neutron number N, the same was done for w.

	Ru	**Ru**	**Cd**	**Cd**	**Pd**	**Pd**	**Mo**	**Mo**	**Kr**	**Kr**	**Se**	**Se**
N	w	σ	w	σ	w	σ	w	σ	w	σ	w	σ
40									12.85	0.03		
42									12.25	0.027	9.39	0.017
44									7.57	0.013	11.05	0.018
46									7.68	0.01	17.94	0.027
48									4.01	0.005	19.12	0.028
50			-49.08	-0.0357					-28.5	-0.018	-46.09	-0.032
52	-9.147	-0.01098	-23.565	-0.0235	-12.862	-0.0149						
54	1.221	0.002	-4.555	-0.006	0.336	0.0005						
56	5.82	0.0107	2.308	0.004	6.922	0.012						
56	5.820	0.011	2.309	0.004	6.922	0.012						
58	5.475	0.012	7.441	0.012	5.220	0.009	1.966	0.004				
60	7.965	0.022	4.075	0.006	8.822	0.017	8.715	0.029				
62	9.001	0.033	1.016	0.002	6.170	0.014	10.959	0.057				
64	10.648	0.044	4.714	0.008	8.176	0.022	11.395	0.066				
66	0.742	0.045	4.744	0.008	8.760	0.025	11.014	0.057				
68	9.833	0.042	3.742	0.007	7.890	0.024						
70			3.993	0.008	9.048	0.027						





^{100}Pd
11
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9
8
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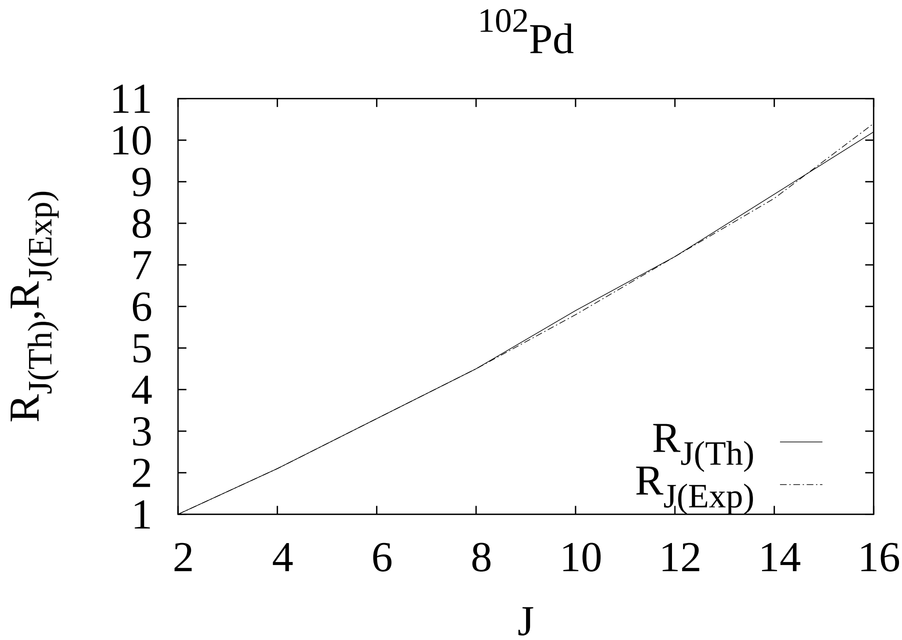
16
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{102}Pd
11
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9
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7
6
5
4
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2
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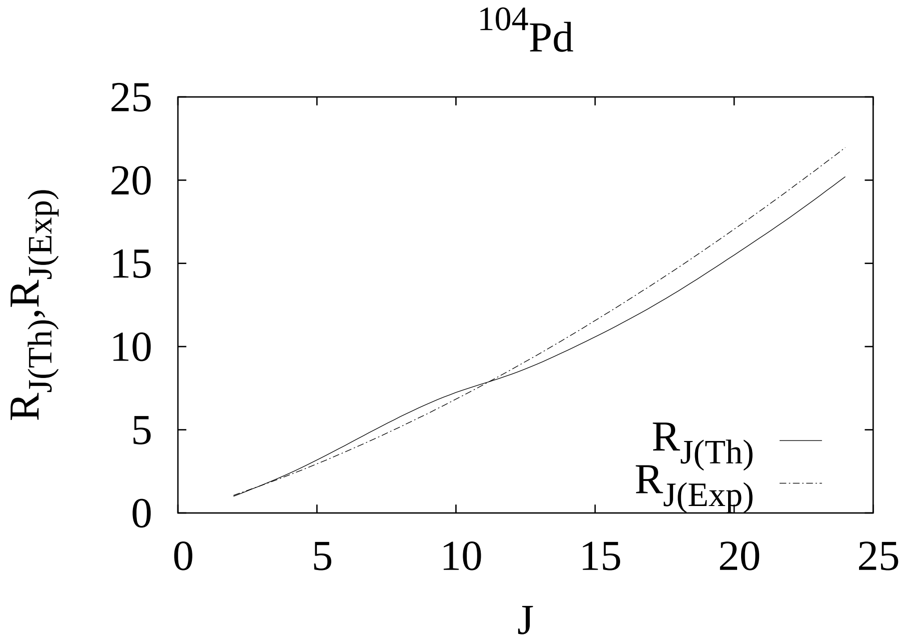
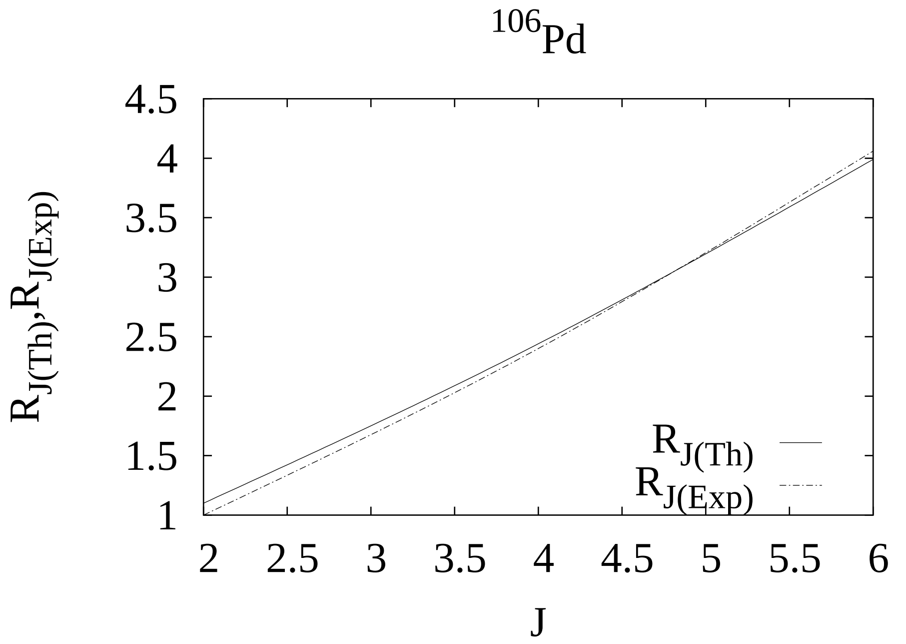
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$

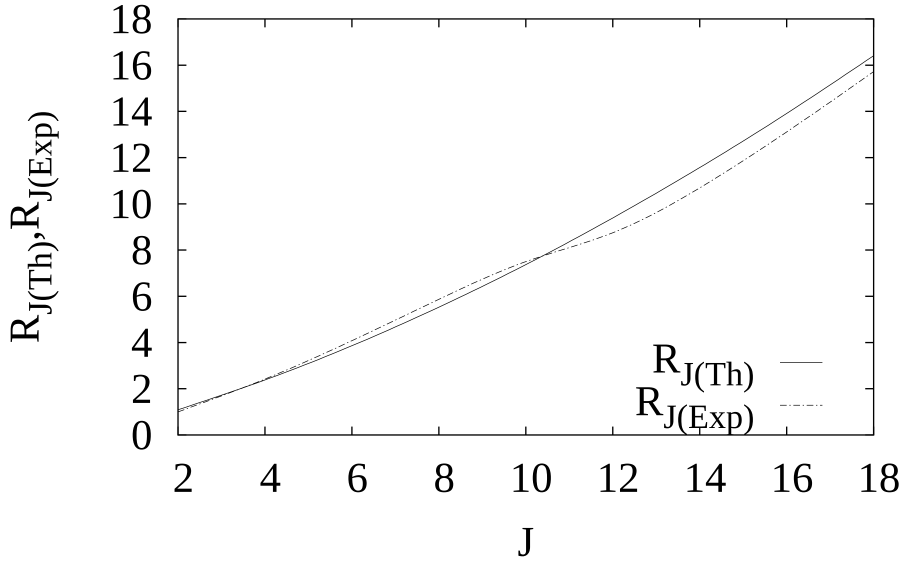


^{104}Pd
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$
0
5
10
15
20
25



^{106}Pd
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6

^{108}Pd



18
16
14
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6
4
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0
2
4
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14
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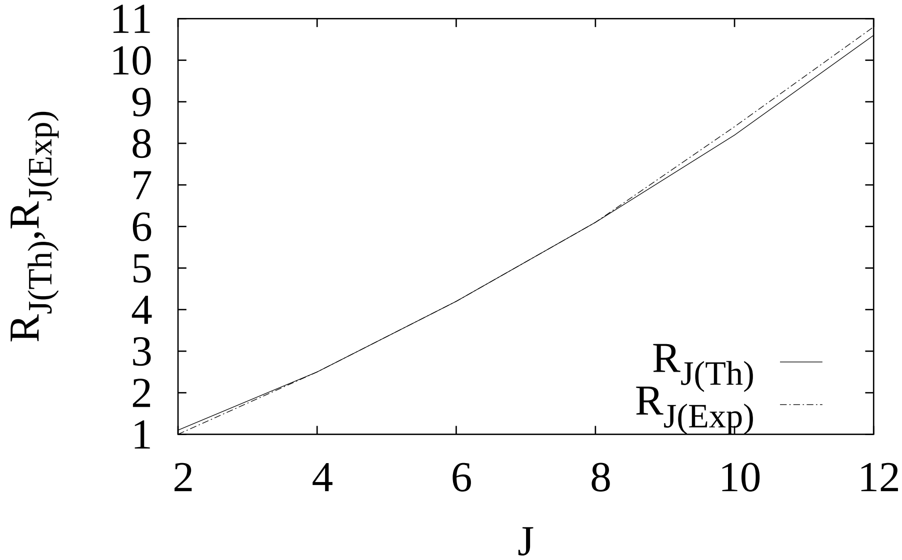
18
J
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)

^{110}Pd



11
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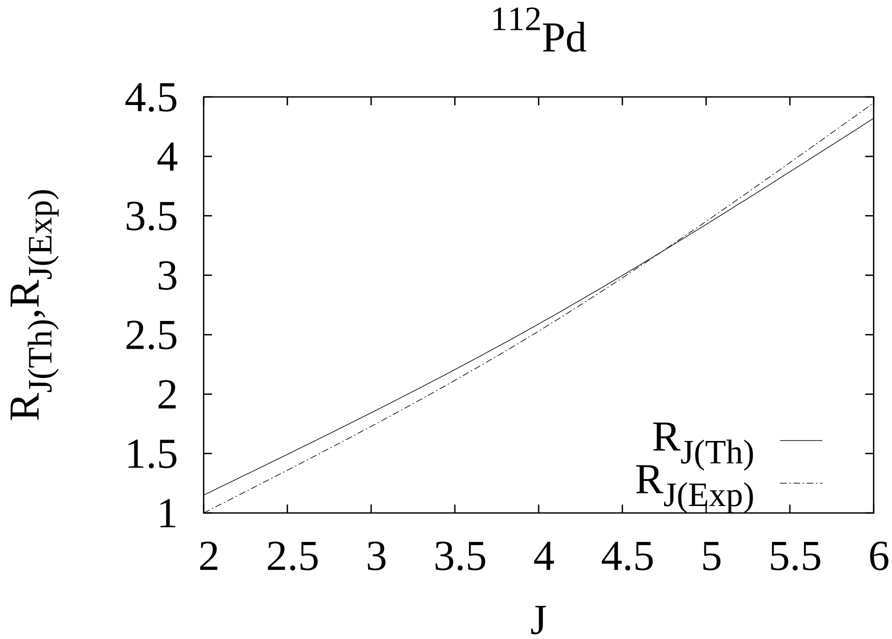
J
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)



^{112}Pd
4.5
4
3.5
3
2.5
2
1.5
1
2
2.5
3
3.5
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4.5
5
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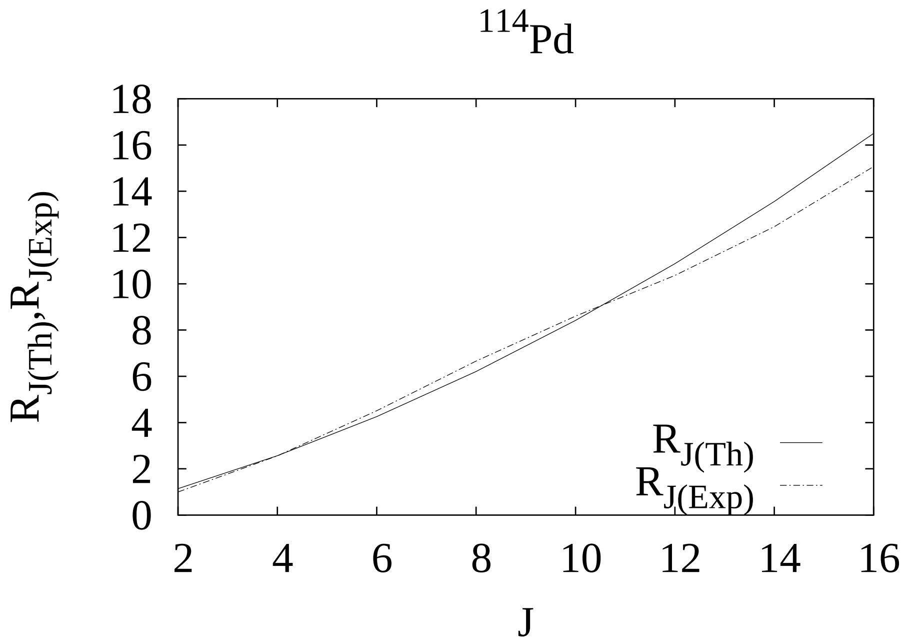
6
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{114}Pd
18
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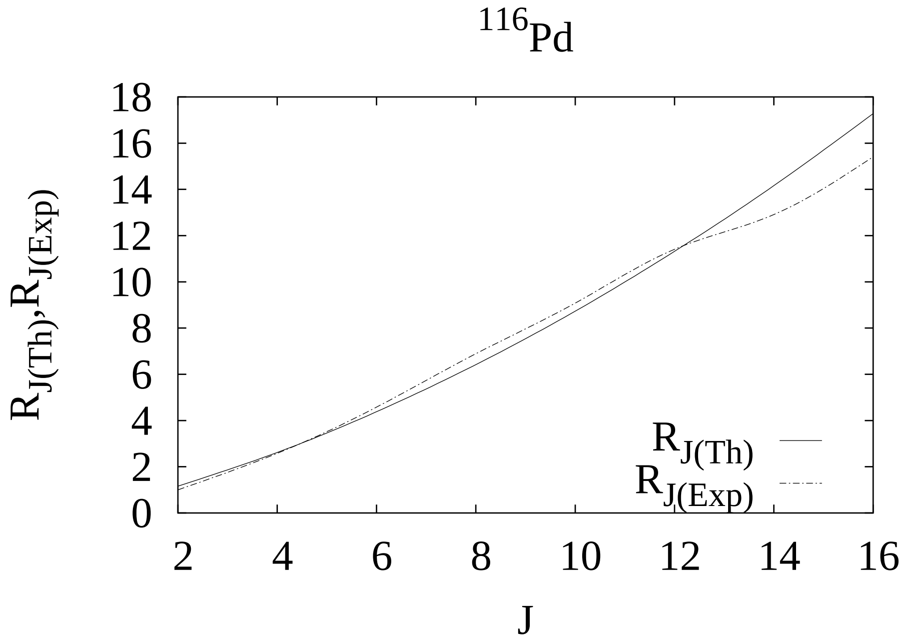
16
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



116Pd
R J(Th),R J(Exp)
18
16
14
12
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8
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0
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4
6
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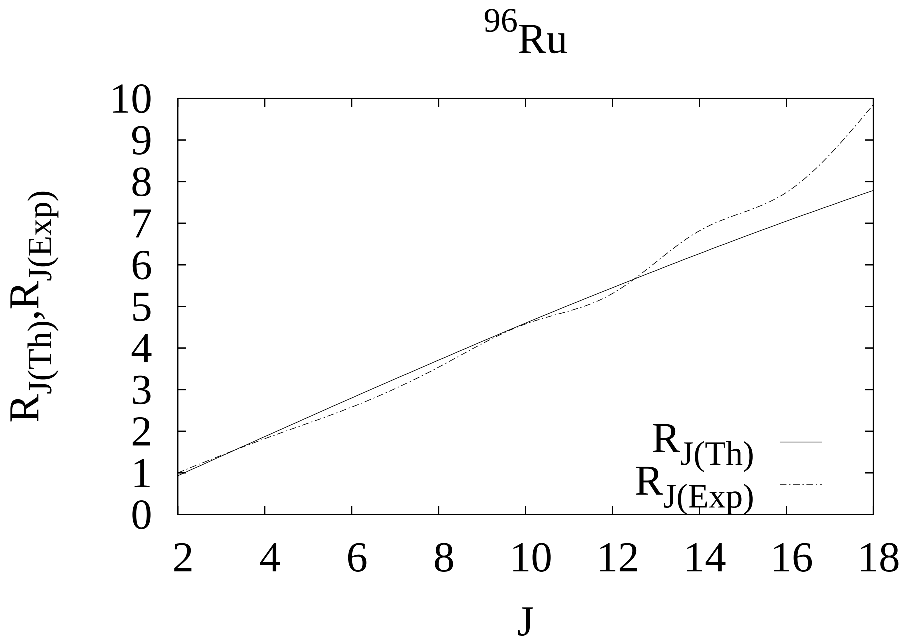
10
12
14
16
J
R J(Th)
R J(Exp)



96Ru
R J(Th),R J(Exp)
10
9
8
7
6
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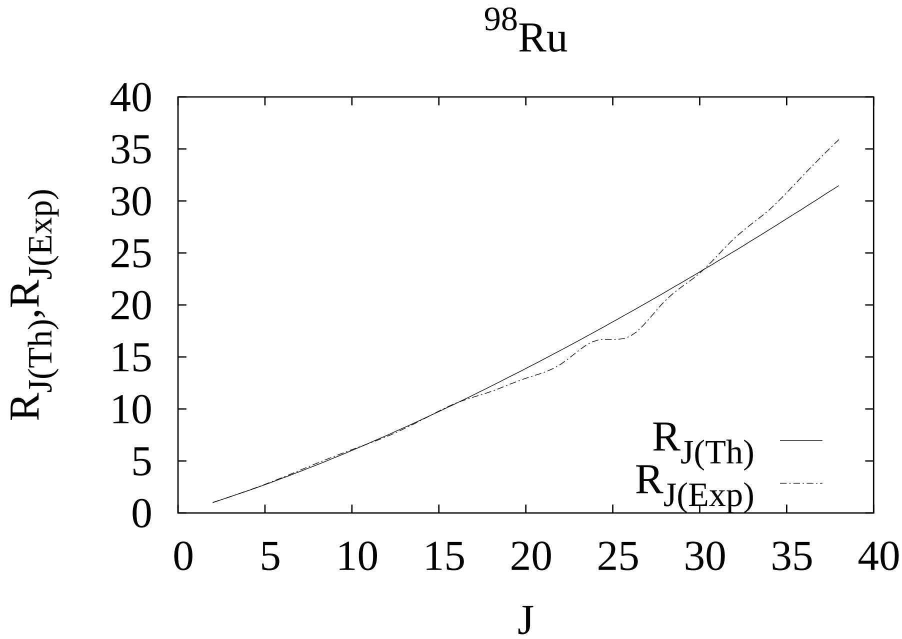
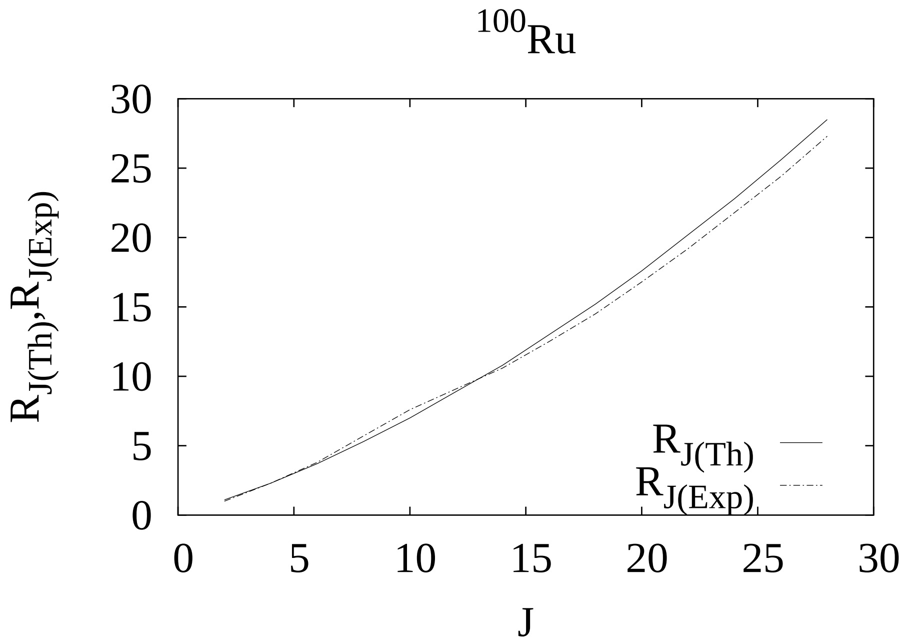
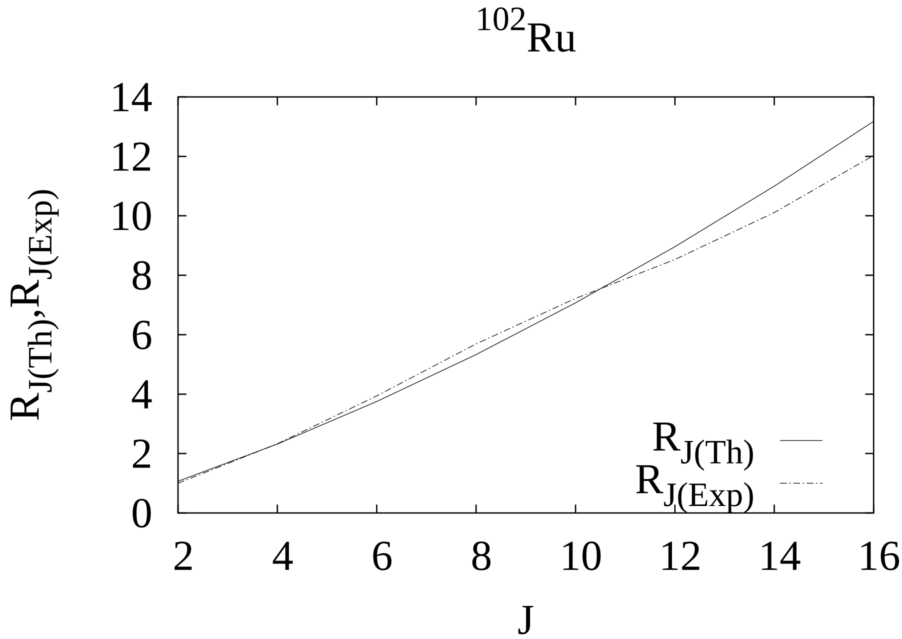
J
R J(Th)
R J(Exp)



^{98}Ru
$R_{J(Th)},R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



^{100}Ru
$R_{J(Th)},R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



102Ru
14
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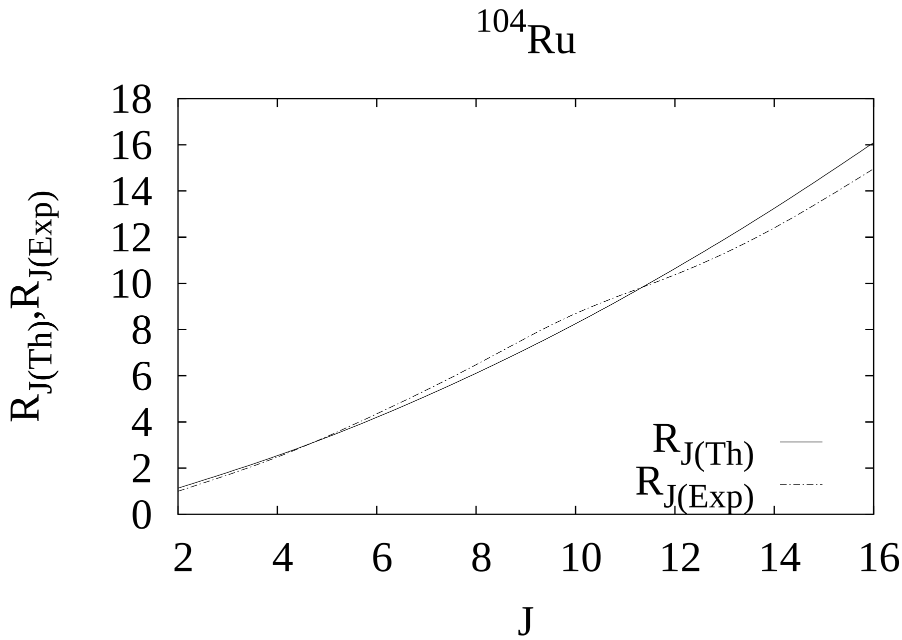
J
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)



104Ru
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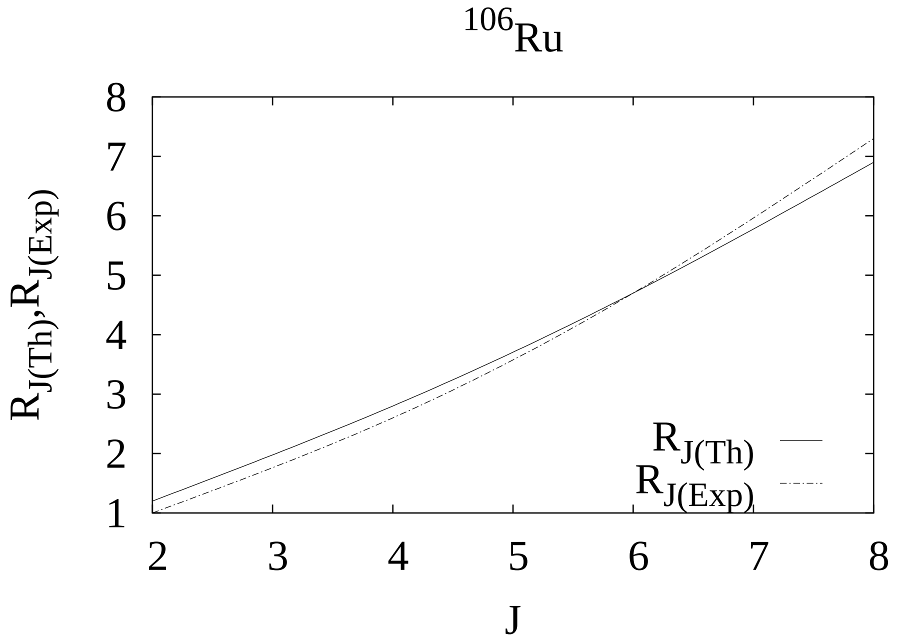
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)



^{106}Ru
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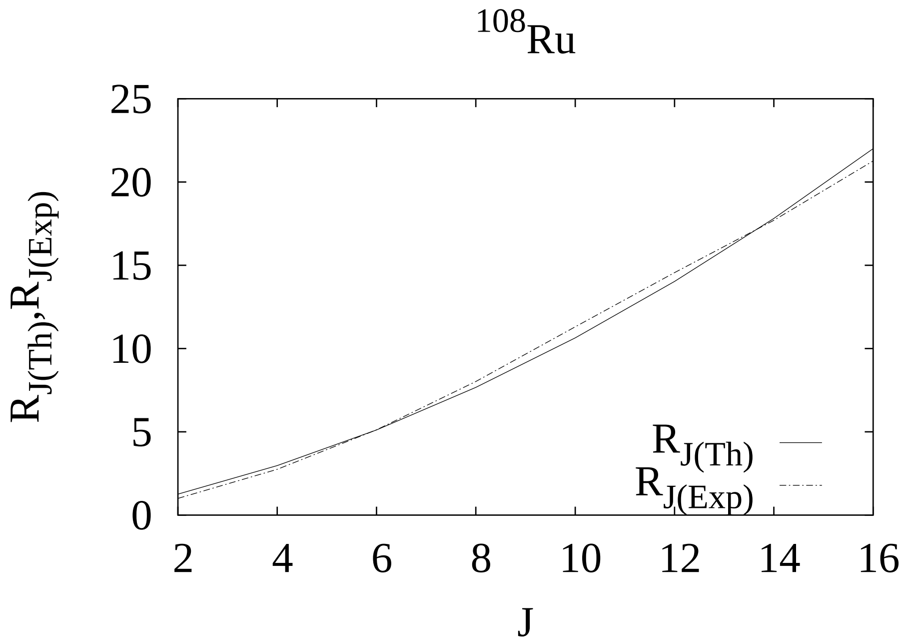
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{108}Ru
25
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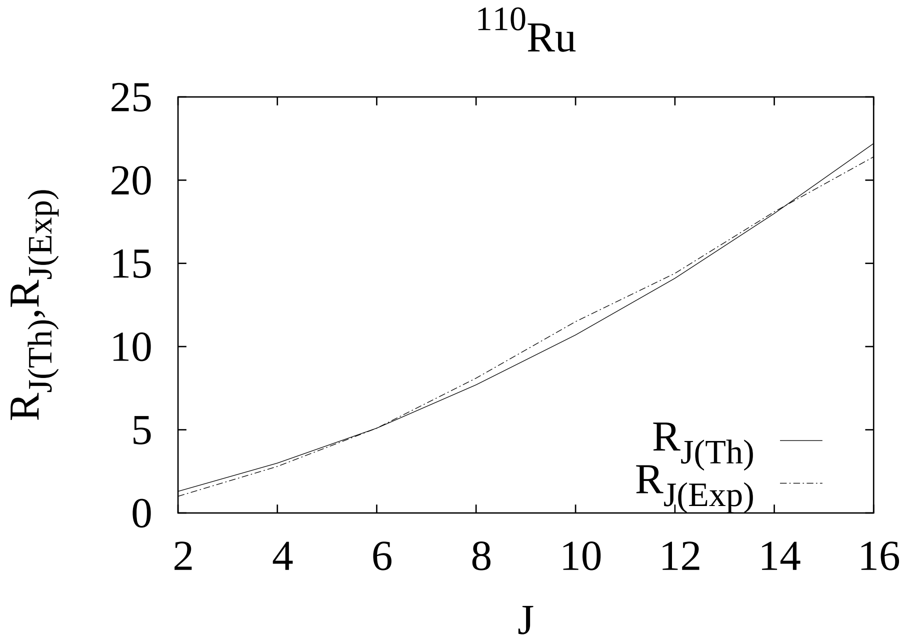
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{110}Ru
$R_{J(Th)},R_{J(Exp)}$
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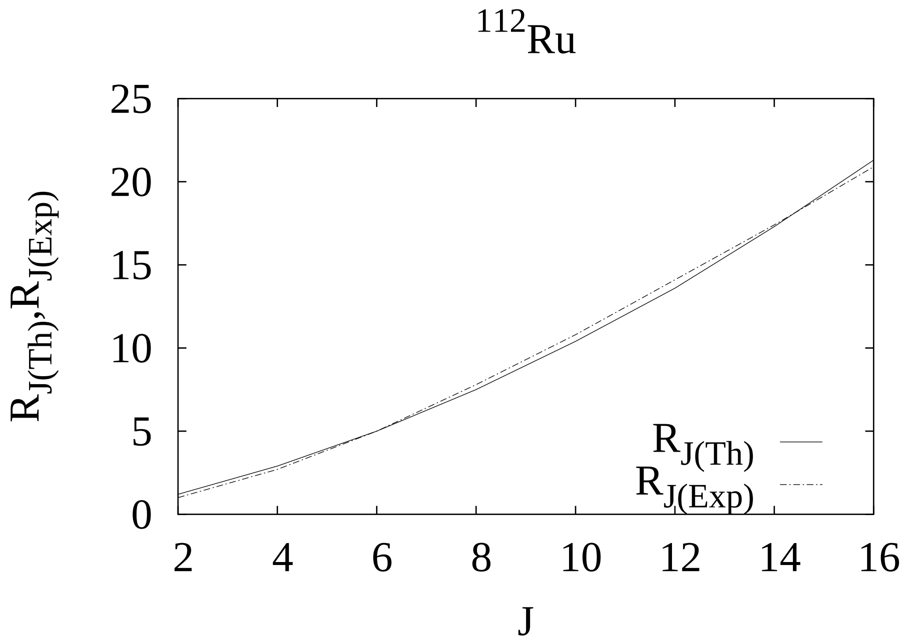
J
$R_{J(Th)}$
$R_{J(Exp)}$



^{112}Ru
$R_{J(Th)},R_{J(Exp)}$
25
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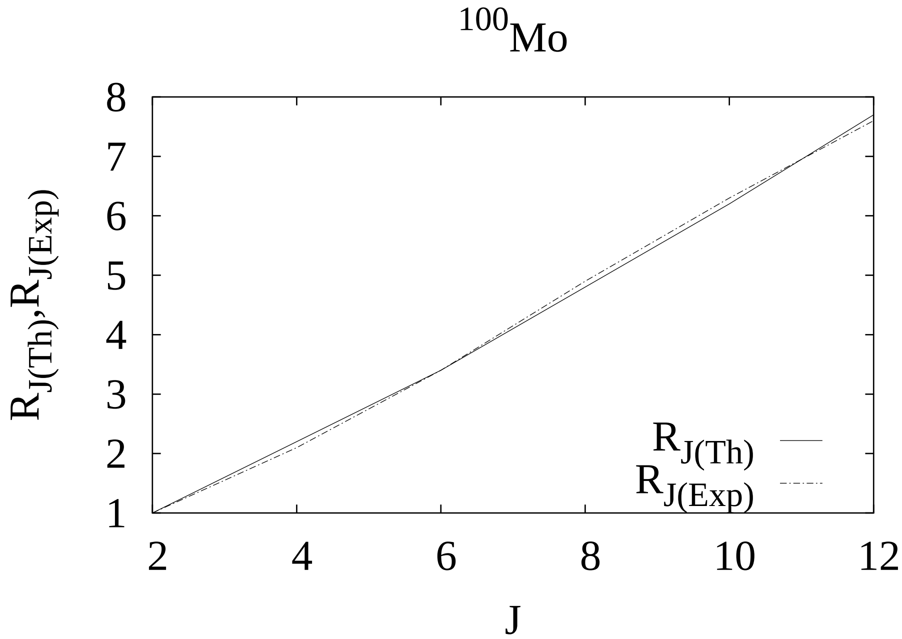
J
$R_{J(Th)}$
$R_{J(Exp)}$



^{100}Mo
8
7
6
5
4
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2
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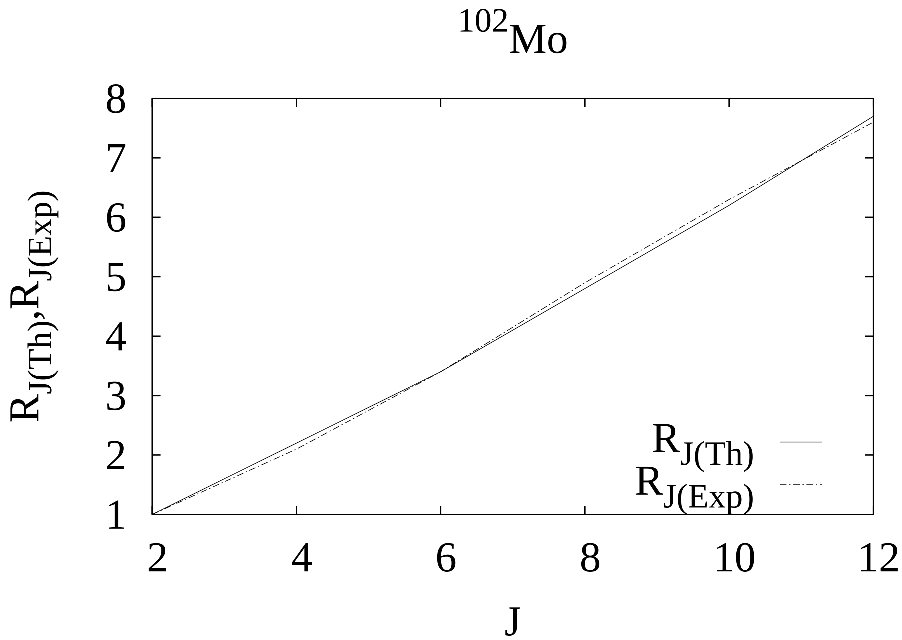
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{102}Mo
8
7
6
5
4
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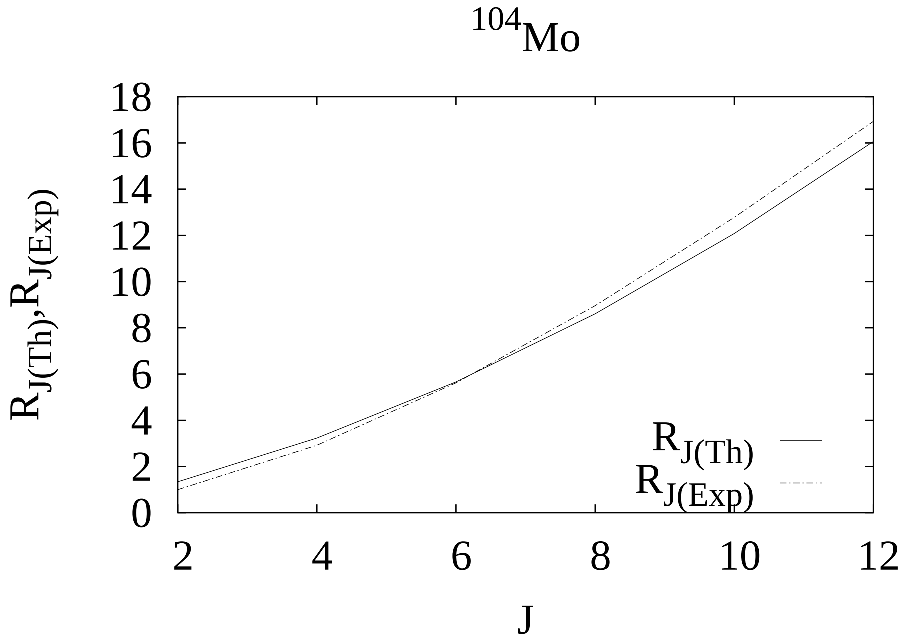
10
12
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



^{104}Mo
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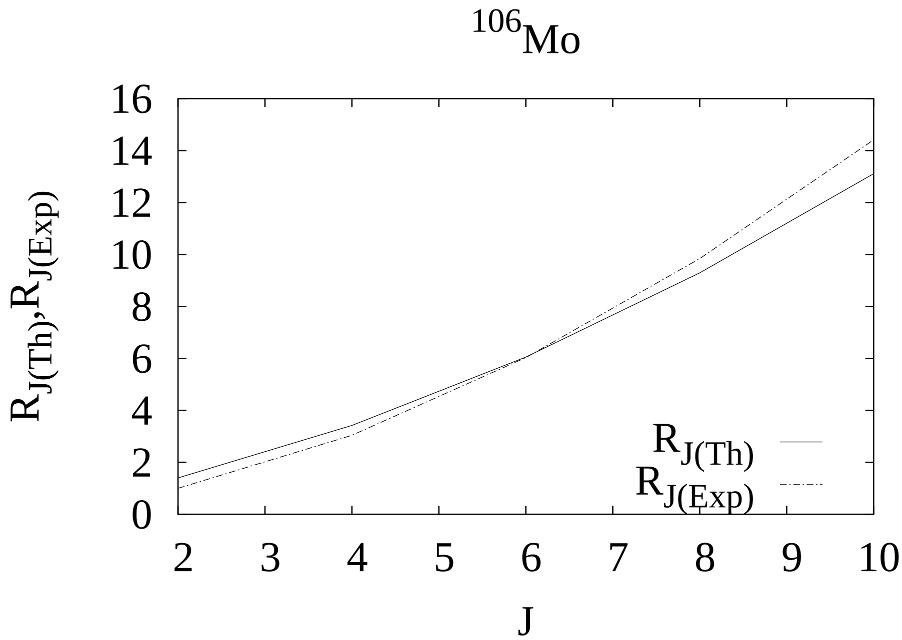
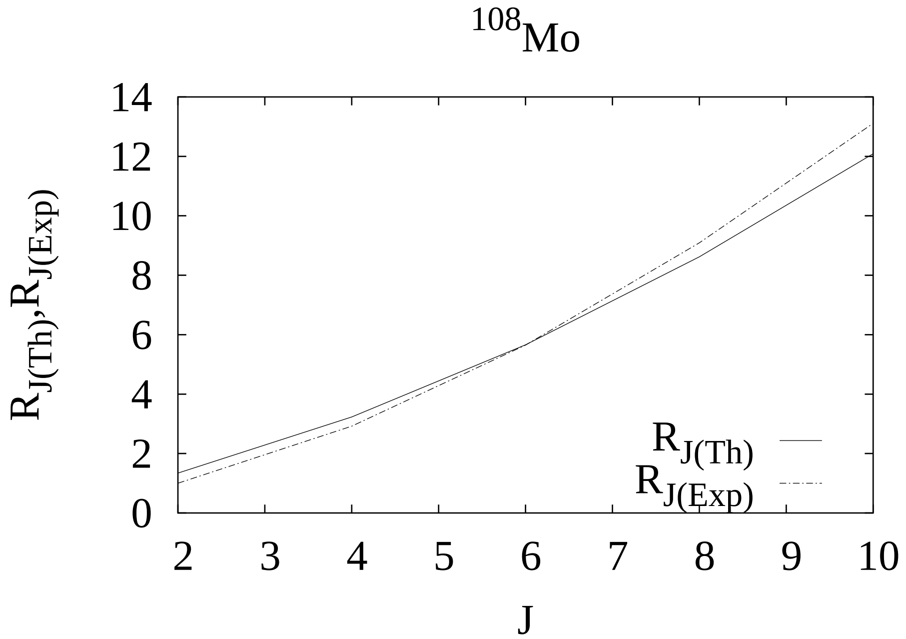
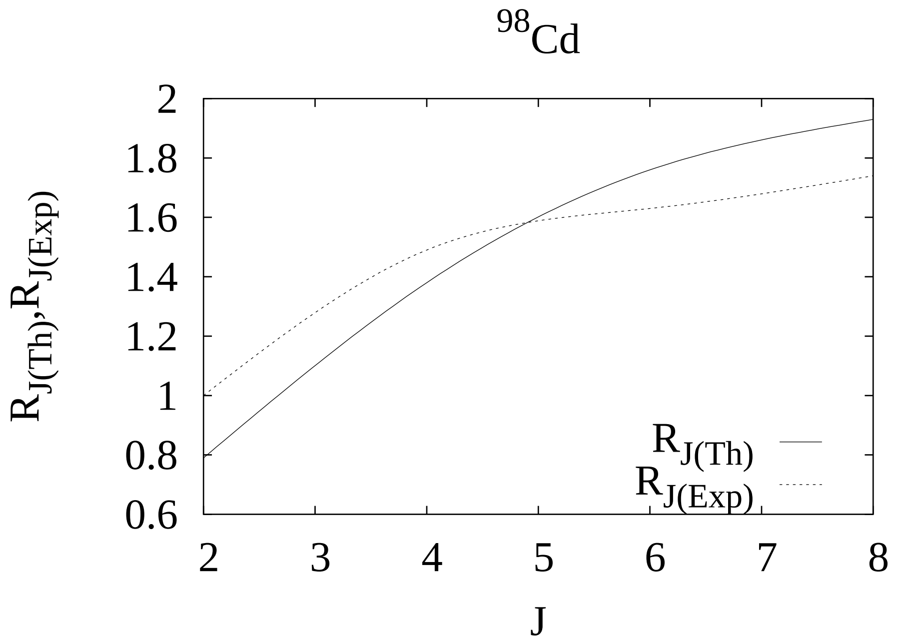
16
14
12
10
8
6
4
2
0
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$
2
4
6
8
10
12
J



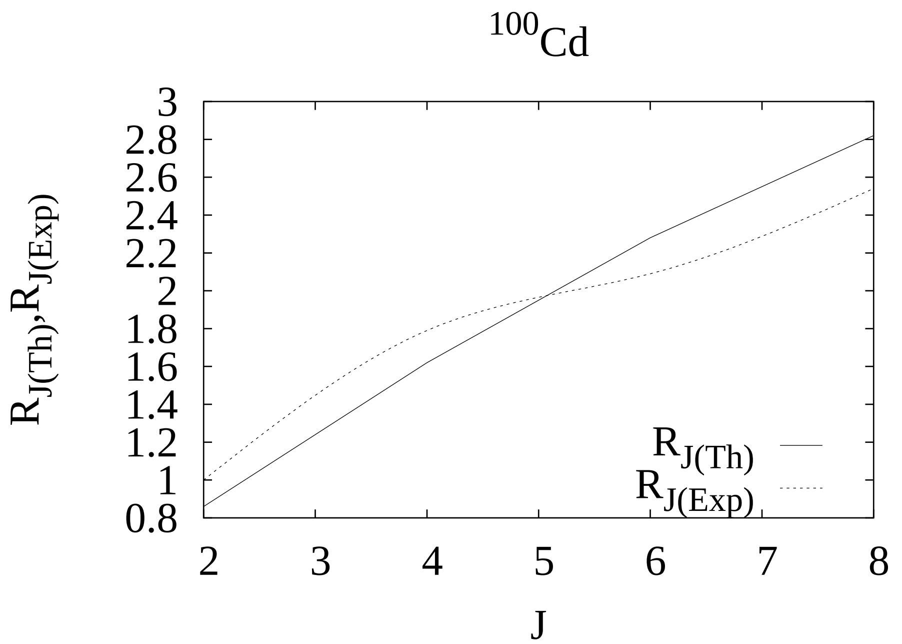
^{106}Mo
16
14
12
10
8
6
4
2
0
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$
2
3
4
5
6
7
8
9
10
J

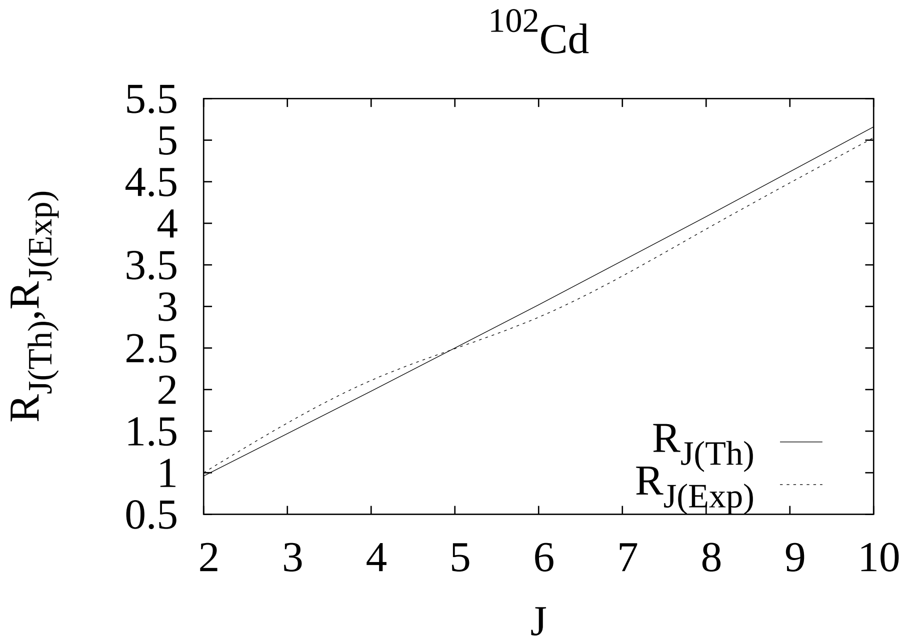


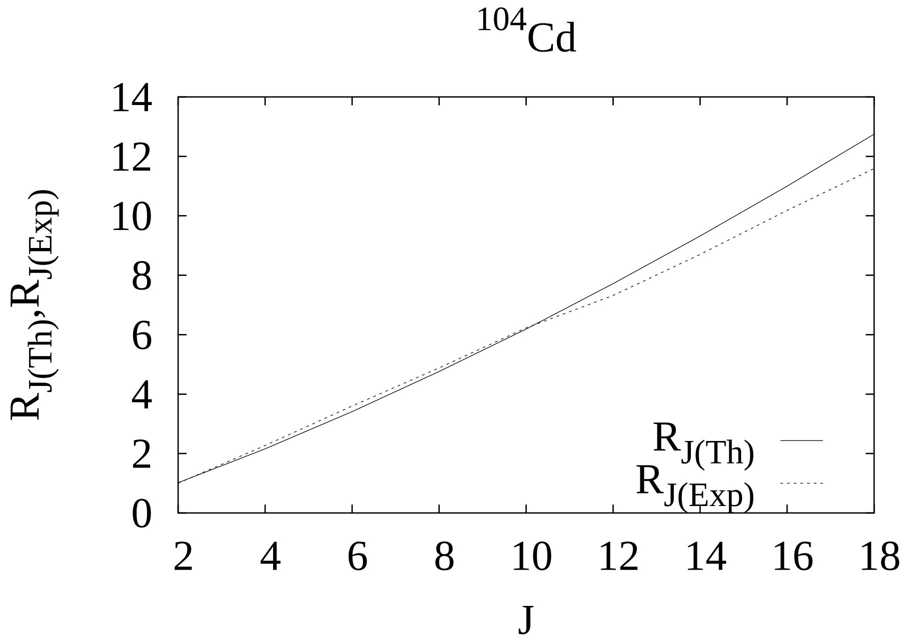
^{108}Mo
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



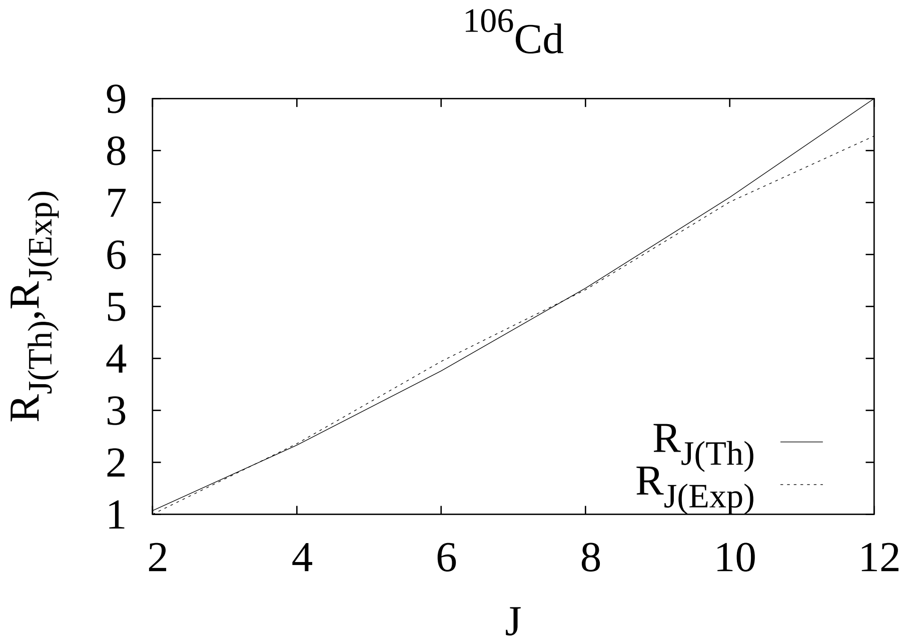
^{98}Cd
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$


^{100}Cd
$R_{J(Th)},R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$
J

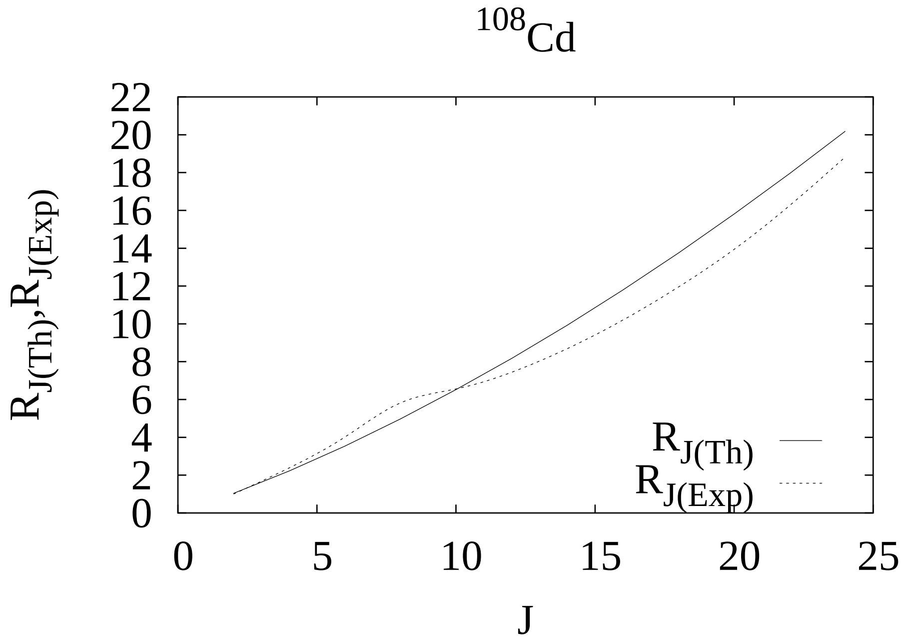

^{102}Cd
$R_{J(Th)},R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$
J



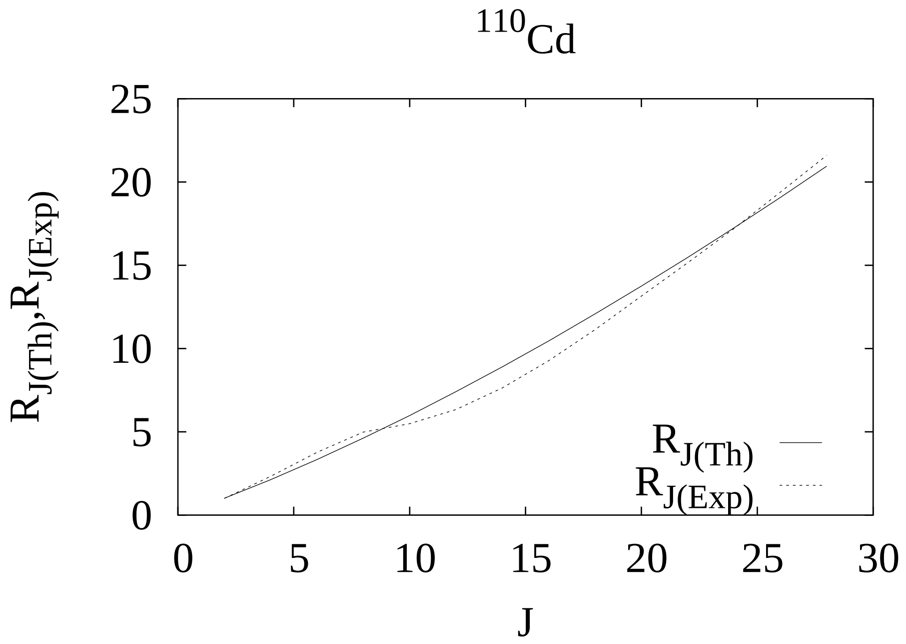
104Cd
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
18
J
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)



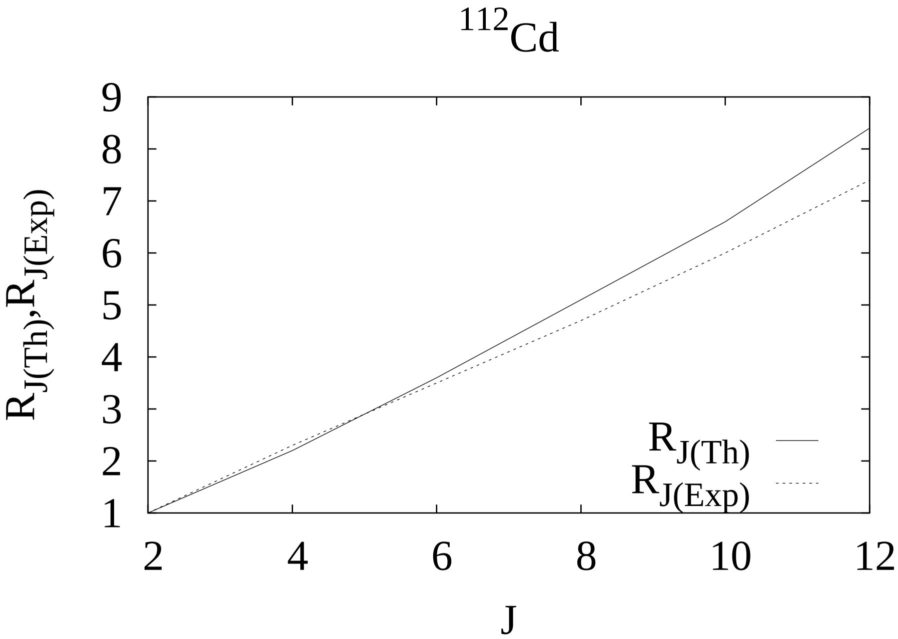
106Cd
9
8
7
6
5
4
3
2
1
2
4
6
8
10
12
J
R_J(Th),R_J(Exp)
R_J(Th)
R_J(Exp)



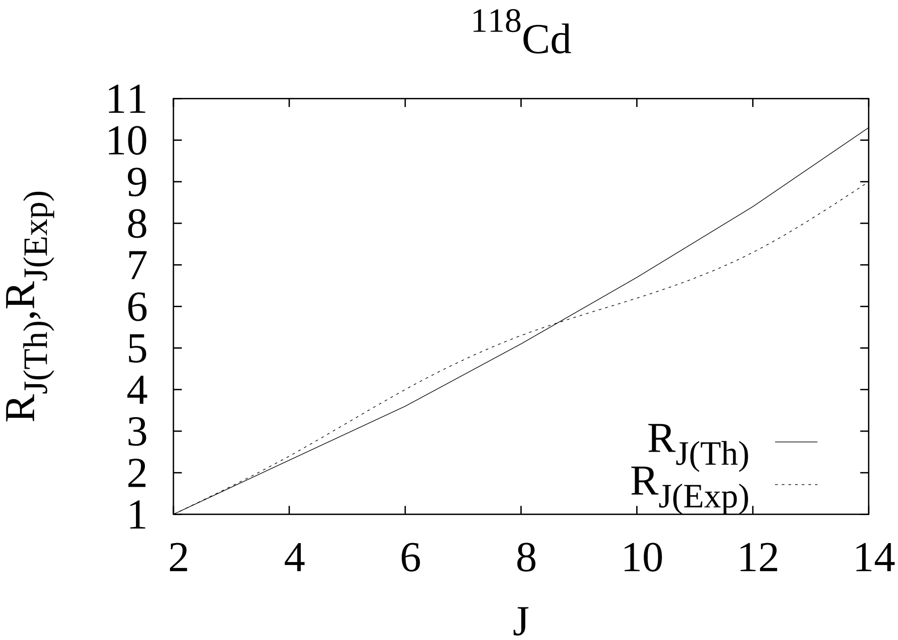
108Cd
RJ(Th),RJ(Exp)
J
RJ(Th)
RJ(Exp)



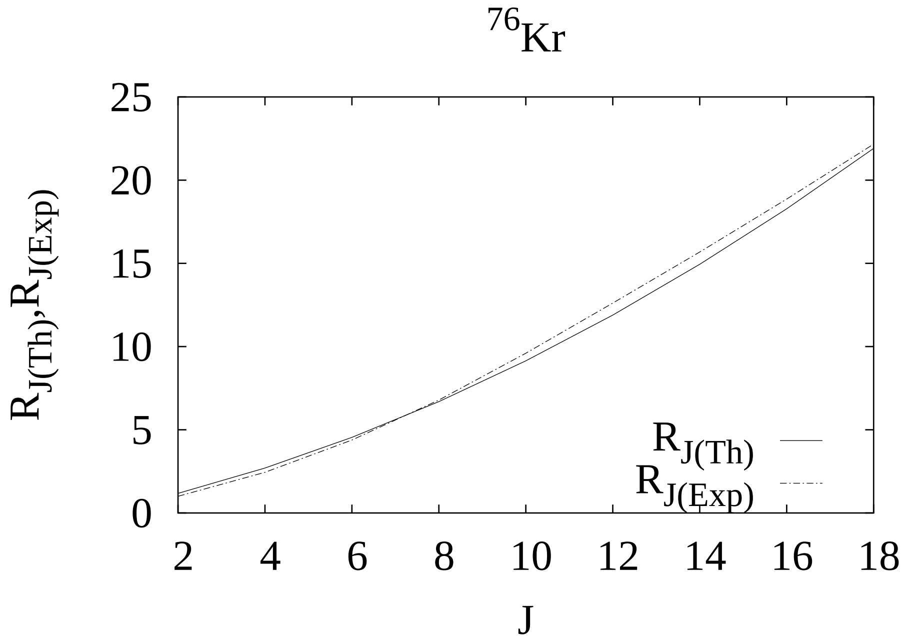
110Cd
RJ(Th),RJ(Exp)
J
RJ(Th)
RJ(Exp)

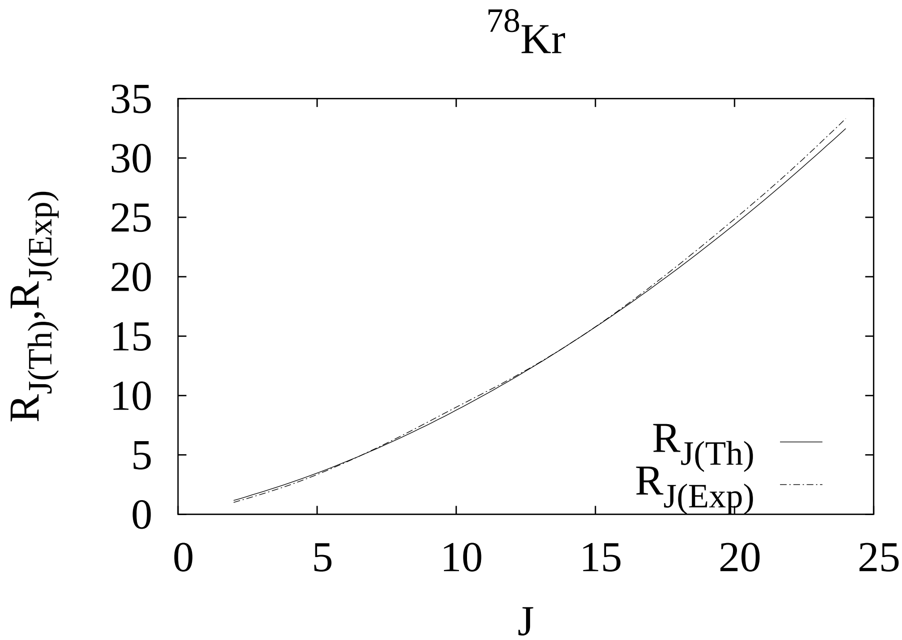


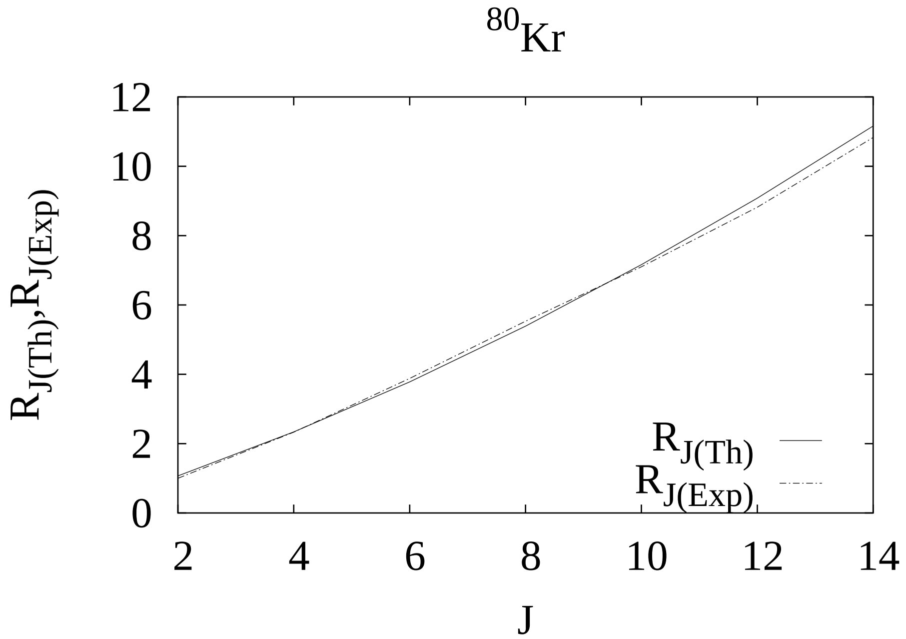
^{112}Cd
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



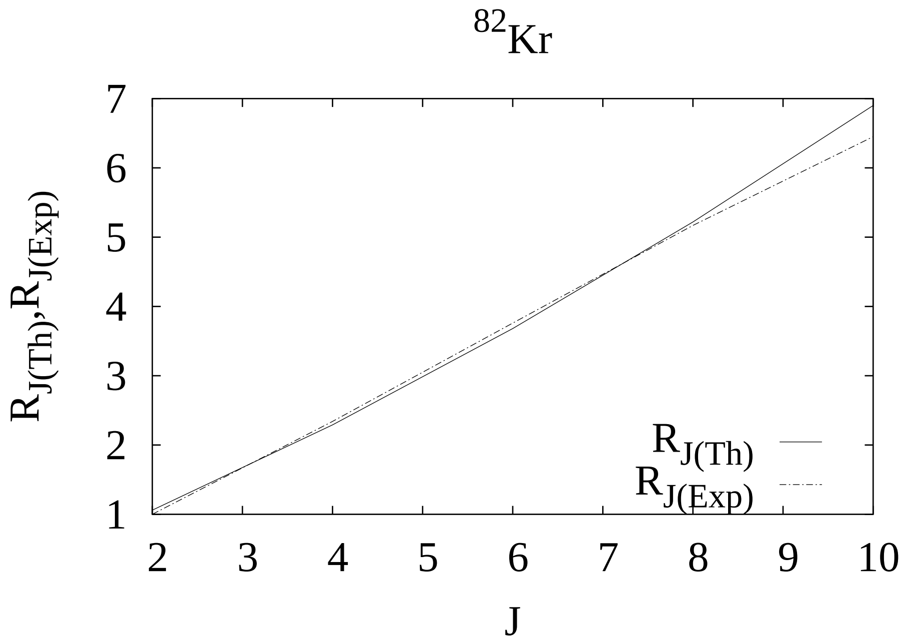
^{118}Cd
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$


76Kr
25
20
15
10
5
0
2
4
6
8
10
12
14
16
18
J
RJ(Th),RJ(Exp)
RJ(Th)
RJ(Exp)

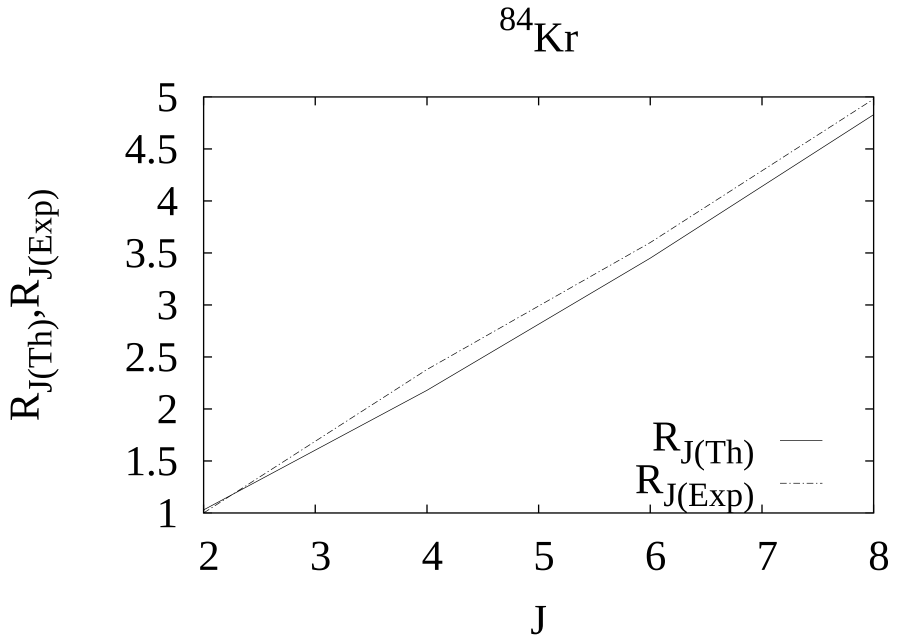

78Kr
35
30
25
20
15
10
5
0
0
5
10
15
20
25
J
RJ(Th),RJ(Exp)
RJ(Th)
RJ(Exp)



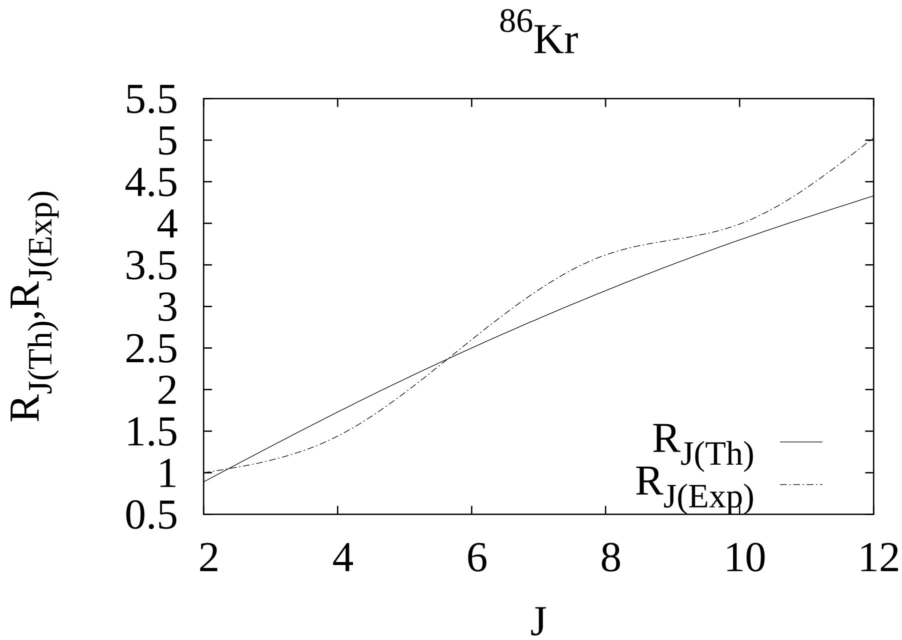
^{80}Kr
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$
0
2
4
6
8
10
12
14



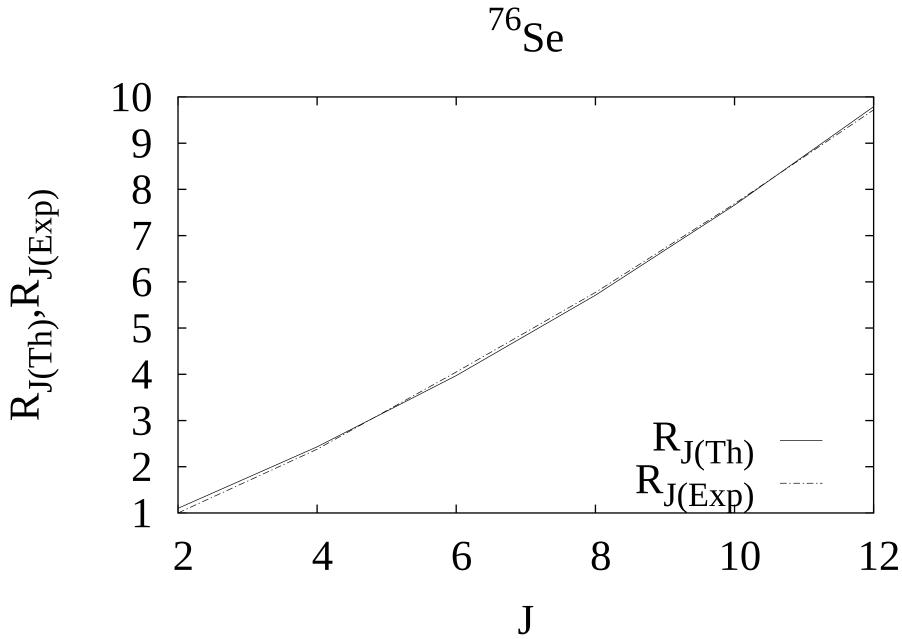
^{82}Kr
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$
1
2
3
4
5
6
7
8
9
10



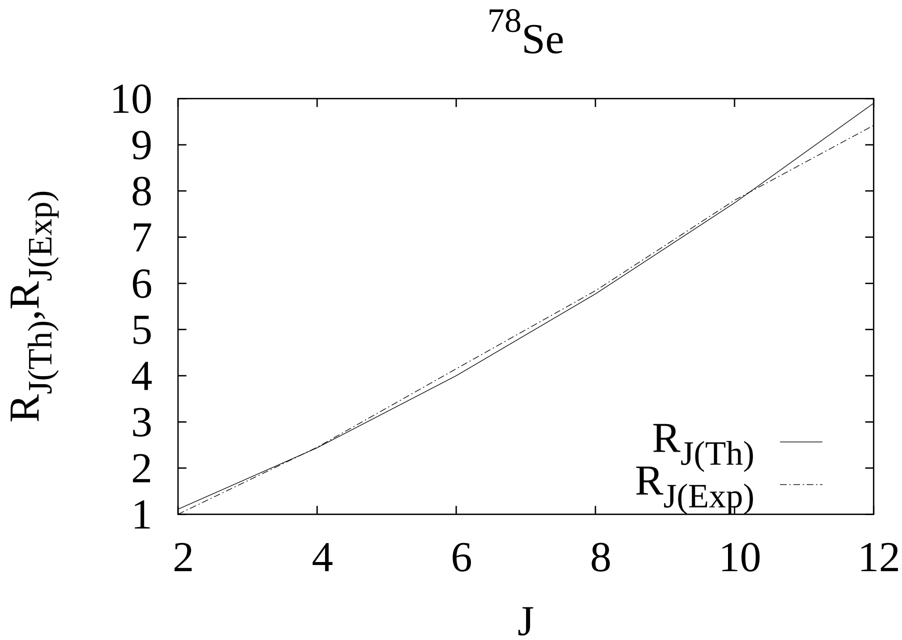
^{84}Kr
5
4.5
4
3.5
3
2.5
2
1.5
1
2
3
4
5
6
7
8
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



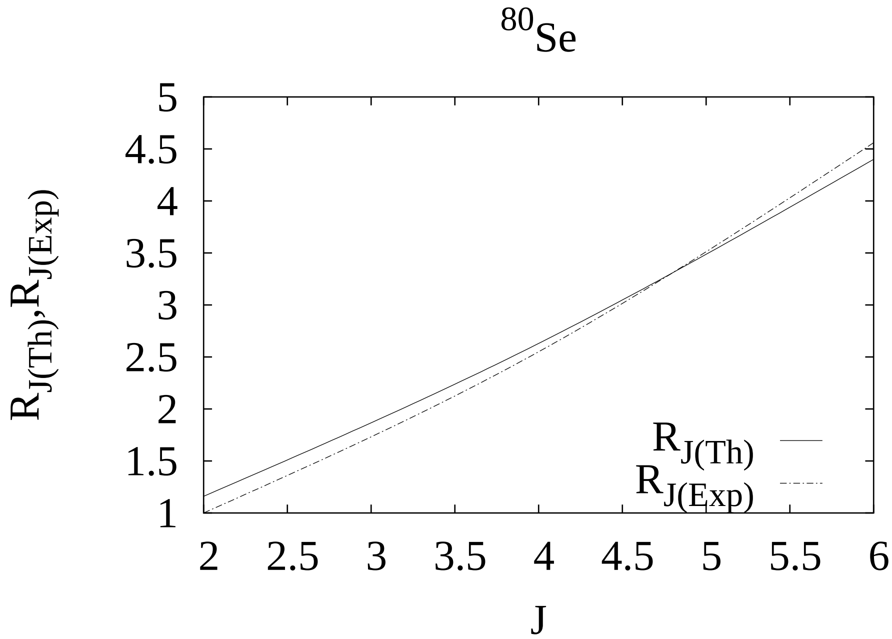
^{86}Kr
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
2
4
6
8
10
12
J
$R_{J(Th)}, R_{J(Exp)}$
$R_{J(Th)}$
$R_{J(Exp)}$



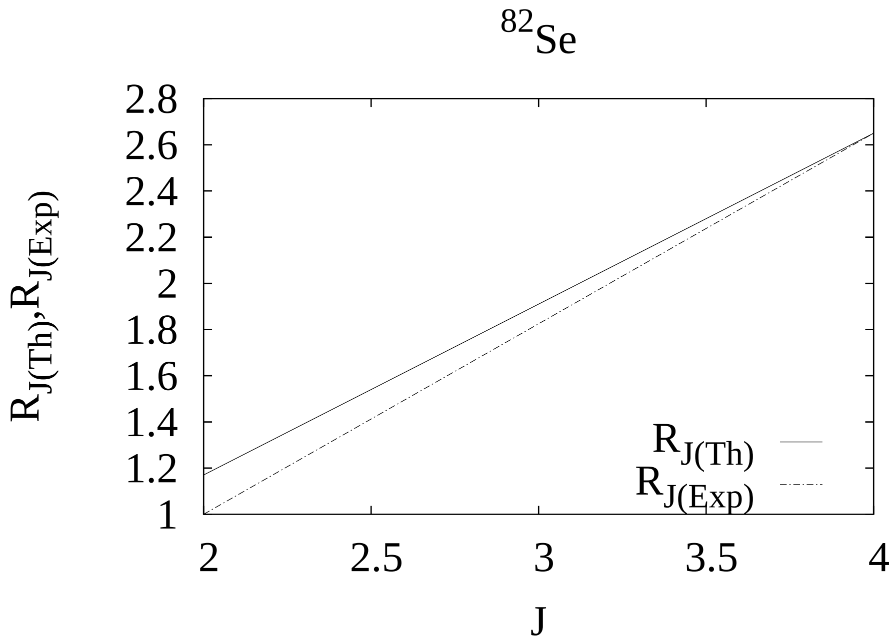
^{76}Se
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



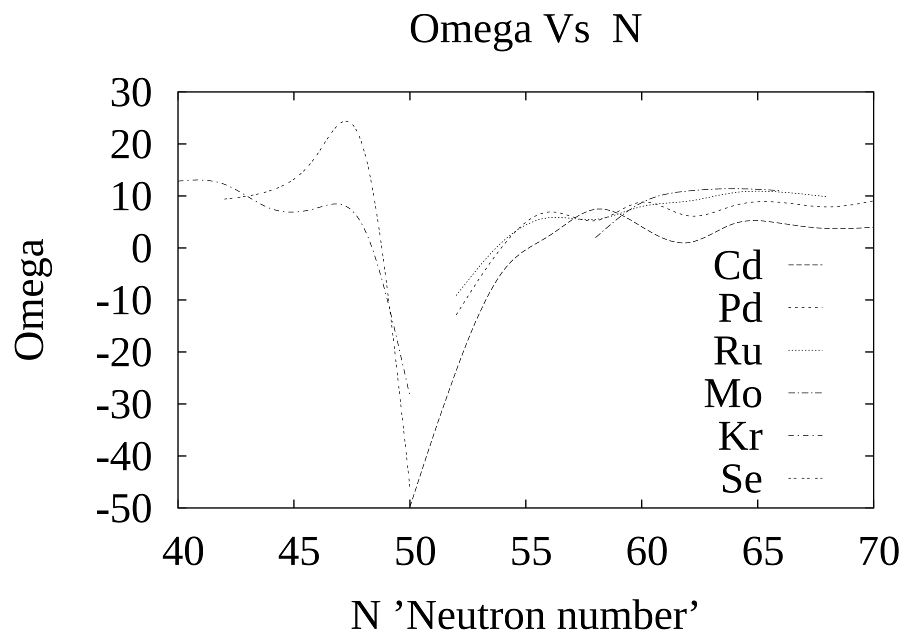
^{78}Se
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$

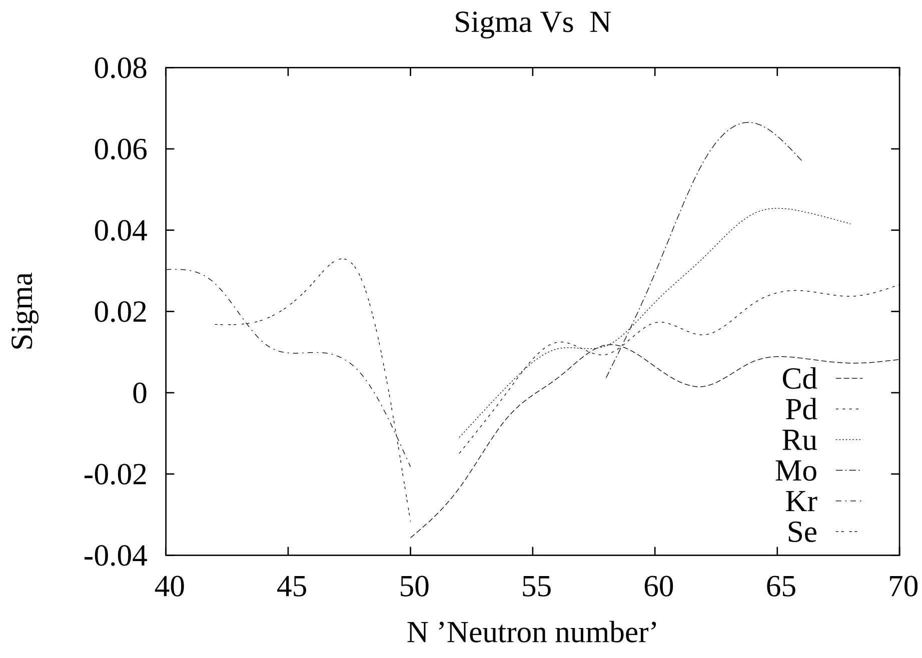


^{80}Se
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$



^{82}Se
$R_{J(Th)}, R_{J(Exp)}$
J
$R_{J(Th)}$
$R_{J(Exp)}$


Omega Vs N
Omega
30
20
10
0
-10
-20
-30
-40
-50
40
45
50
55
60
65
70
N 'Neutron number'
Cd
Pd
Ru
Mo
Kr
Se


Sigma Vs N
Sigma
0.08
0.06
0.04
0.02
0
-0.02
-0.04
40
45
50
55
60
65
70
N 'Neutron number'
Cd
Pd
Ru
Mo
Kr
Se

For the transition from the state with one pair of coupled bosons $E(0_2^+)$, to the state with one uncoupled pair $E(2_1^+)$, the energy ratio R_0, as mentioned earlier, has the value of **2.553** for strong anharmonicity limit, the experimental observed values for such transition are given in the table below.

N	Ru	Cd	Pd	Mo	Kr	Se
40					1.82	
42					2.24	2.01
44					2.14	2.44
46					1.92	2.24
48					2.08	2.15
50					1.74	1.35
52					3.58	
56	2.09		2.86			
58		2.82	2.39	1.29		
60	2.76	2.72	2.83	2.35		
62	3.66	2.24	2.43	4.61		
64		1.98	2.53	5.57		
66		2.5	2.553			
68		2.64	3.35			
70			3.26			

For the transition from the state $E(2_2^+)$ to the ground state $E(0_1^+)$ The energy ratio R has the value **3.78**, the experimental values are given in the table below.

N	Ru	Cd	Pd	Mo	Kr	Se
40					3.98	
42					3.86	3.20
44						3.25
46					3.19	
56	3.43		3.49			
58	3.32	3.74	3.22			
60			3.05			
64		2.38				
66		3.79				
68		3.93				

The energy ratio between the differences in the energies between the states $E(2_2^+)$ and $E(0_2^+)$, and the states $E(2_1^+)$ and $E(0_1^+)$ is equal to **1.223**.

i.e

$$R = \frac{E(2_2^+) - E(0_2^+)}{E(2_1^+) - E(0_1^+)} = 1.223$$

The experimental values are given in the table below.

N	**Ru**	**Cd**	**Pd**	**Mo**	**Kr**	**Se**
40					2.16	
42					1.62	1.20
44						0.81
46					1.28	
56	1.36		0.63			
58	1.34	0.91	0.83			
60			0.84			
64		0.40				
66		1.3				
68		1.29				