March 18

Induction and Inductance

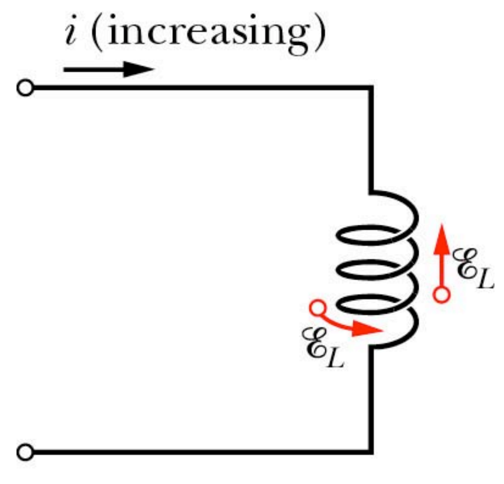
Chapter 31

Review - Self Inductance

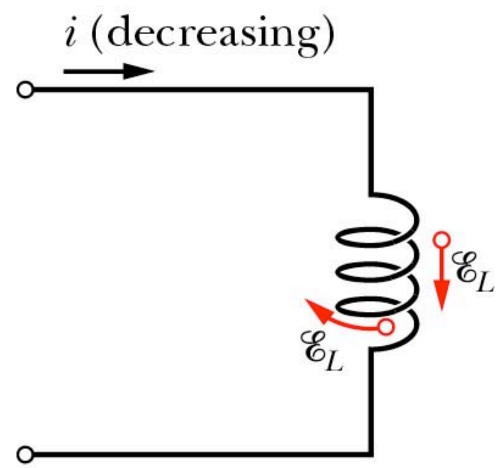
- Self-induce emf, $\mathcal{E}_L$ appears in any coil in which the current is changing

$$\mathcal{E}_L = -L\frac{di}{dt}$$

- Direction of $\mathcal{E}_L$ follows Lenz's law and opposes the change in current



(a)



(b)

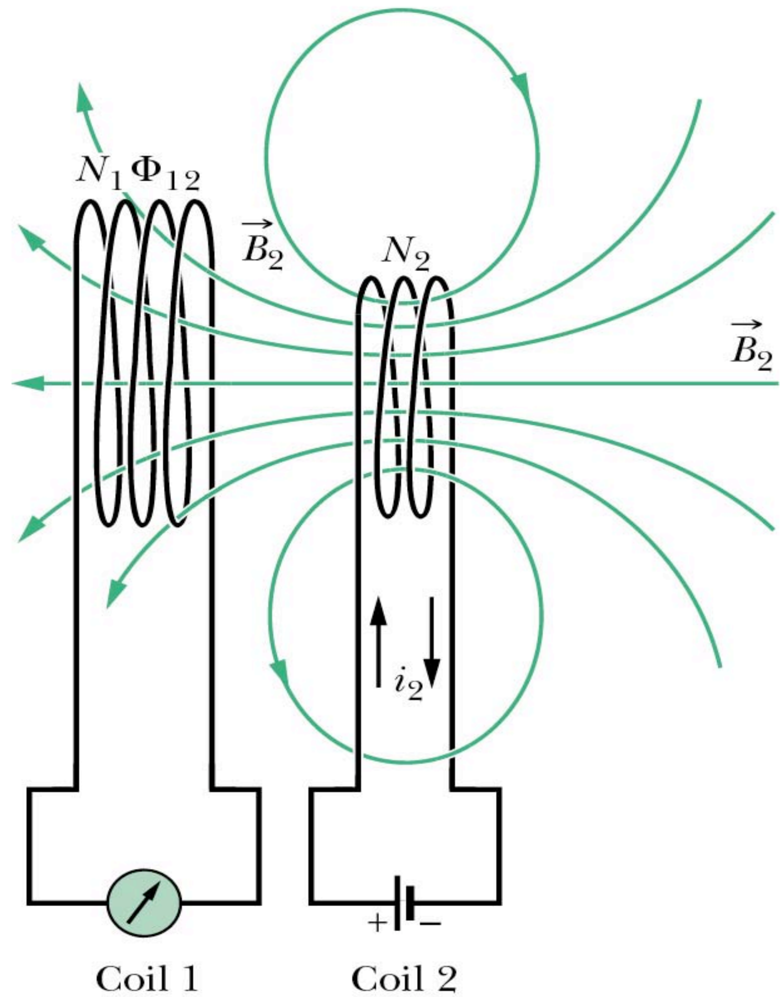
Review - Mutual Inductance

- What is induced emf in coil 1 from a changing current in coil 2?

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

where

$$M = \frac{N_1 \Phi_{12}}{i_2} = \frac{N_2 \Phi_{21}}{i_1}$$



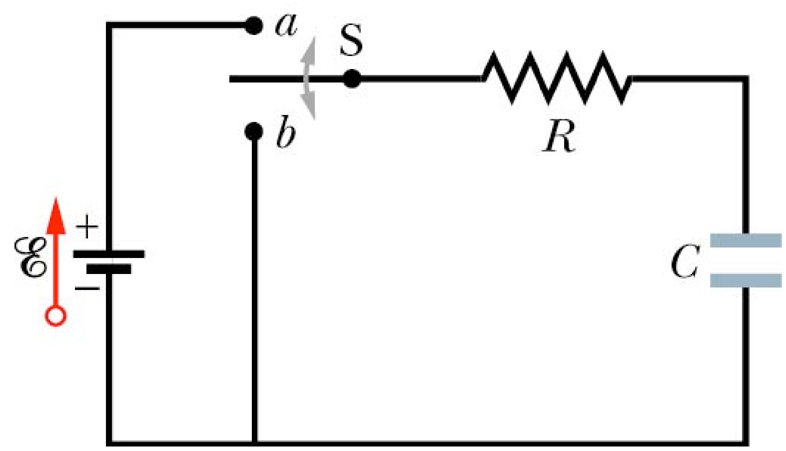
Review: RC circuit

- **RC circuit** is a resistor and capacitor in series
  - Charging up a capacitor (switch at a)
  - Kirchhoff rule for loop:

$$\frac{dq}{dt}R+\frac{q}{C}-\mathscr{E}=0$$

    (=differential equation)
  - Solution

$$q=C\mathscr{E}(1-e^{-t/\tau_c})$$



  - Discharging capacitor (switch at b)

$$q=q_0e^{-t/\tau_c}$$

where $\tau_C = RC$

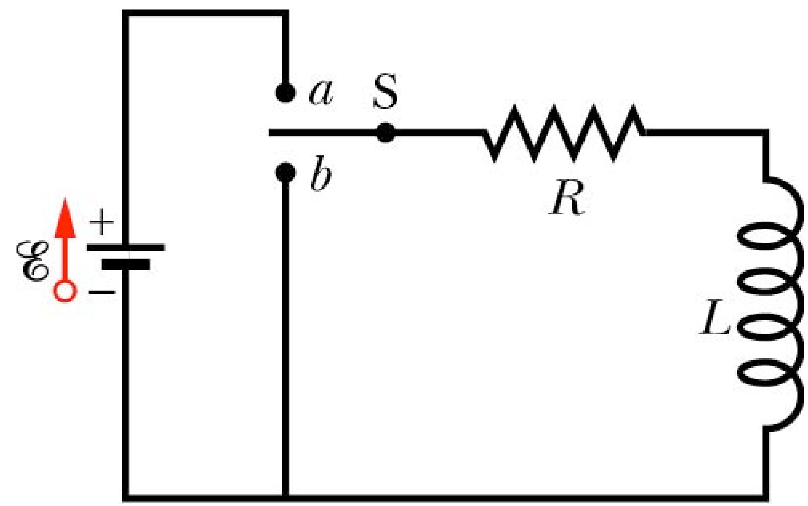
Inductance

- **RL circuit** is a resistor and inductor in series
- Close switch to point *a*
  - **Initially** *i* is increasing through inductor so $\mathcal{E}_L$ opposes rise and *i* through *R* will be

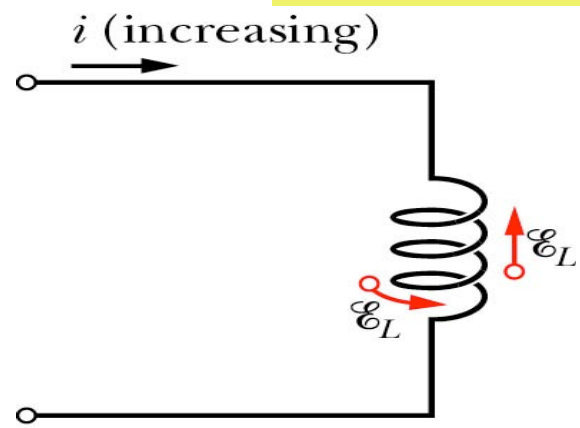
    $$i < \mathcal{E}/R$$

  - **Long time later**, *i* is constant so $\mathcal{E}_L=0$ and *i* in circuit is

    $$i = \mathcal{E}/R$$



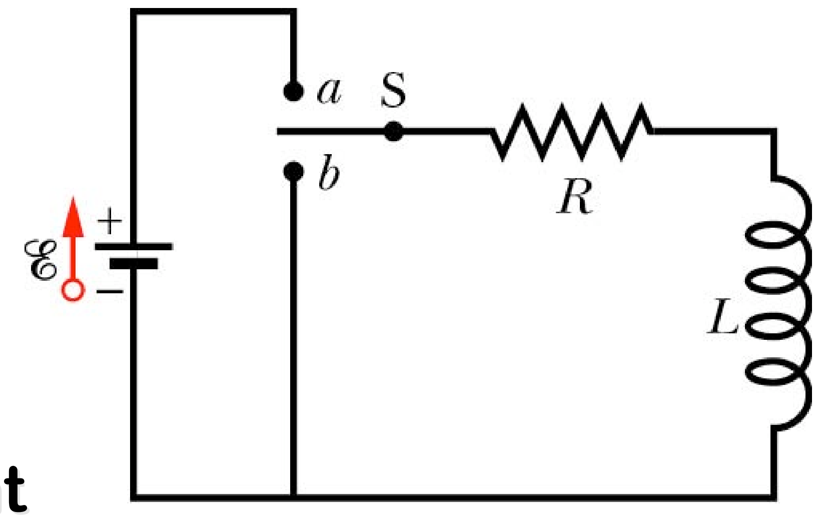
$$\mathcal{E}_L = -L\frac{di}{dt}$$

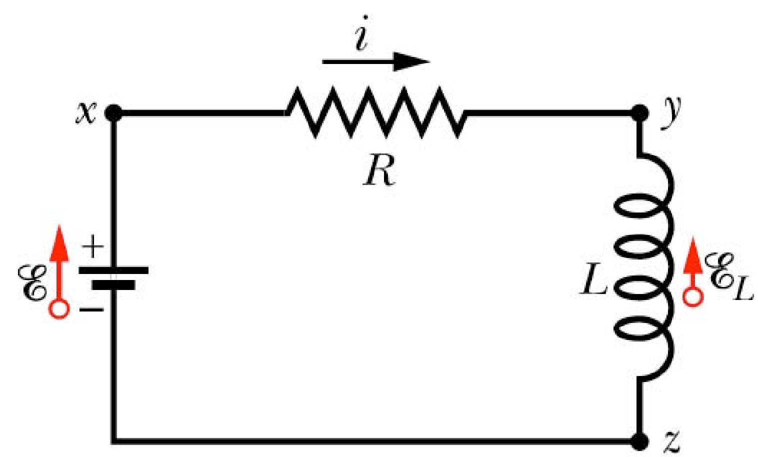


RL Circuit - Differential Equation

- Initially an inductor acts to oppose changes in current through it
- Long time later inductor acts like ordinary conducting wire
- Apply Kirchhoff loop rule right after switch has been closed at *a*

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$





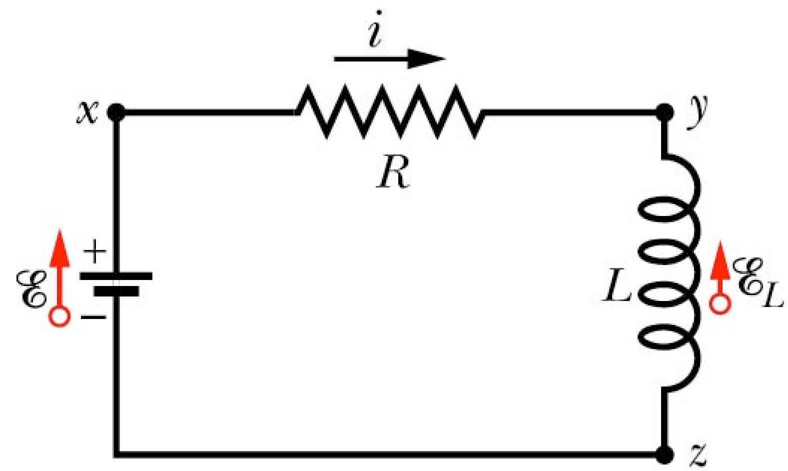
RL Circuit Solution

- Differential equation .... similar to RC circuit:

$$L\frac{di}{dt}+iR-\mathcal{E}=0 \qquad \frac{dq}{dt}R+\frac{q}{C}-\mathcal{E}=0$$

- Solution is

$$i=\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)$$

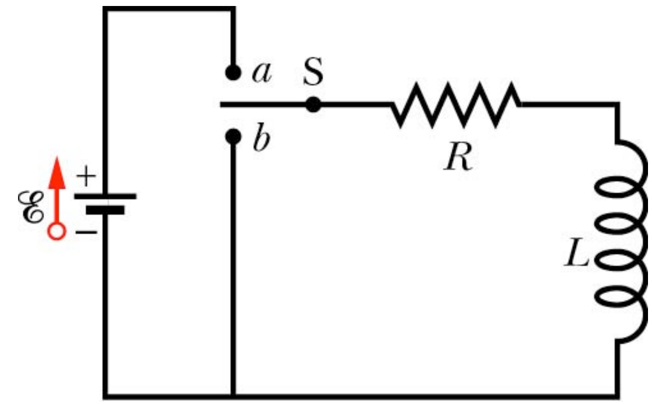


- **Inductive time constant** is

$$\tau_L=\frac{L}{R}$$

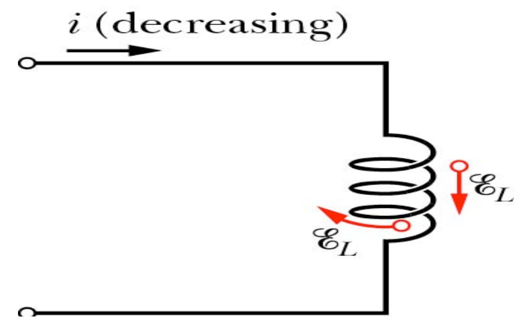
- Satisfies conditions:
  - At t=0, $i = 0$
  - At t=∞, $i = \mathcal{E}/R$

"Discharging"=Stop Current



- Now move switch to position b so battery is out of system
- Current will decrease with time and loop rule gives

$$iR + L\frac{di}{dt} = 0$$



- Solution is

$$i = \frac{\varepsilon}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

- Satisfies conditions
  - At t=0, $i = i_0 = \mathcal{E}/R$
  - At t=∞, $i = 0$

RL circuits Summary

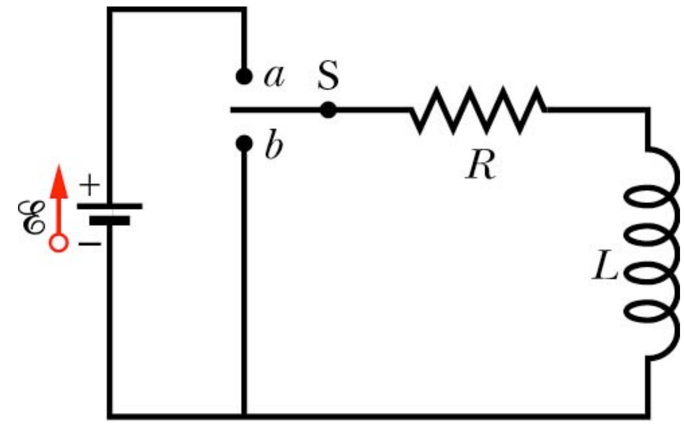
- Circuit is closed (switch to "*a*")

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

- Circuit is opened (switch to "*b*")

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

- Time constant is $\tau_L = \frac{L}{R}$



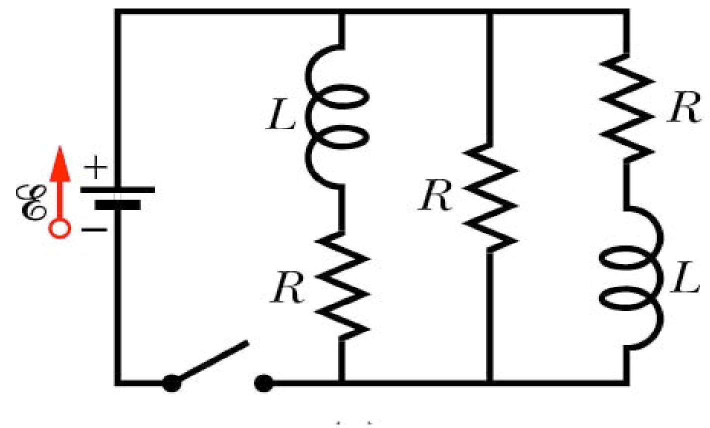
Switch at *a*, current through inductor is:

> Initially $i = 0$ (acts like broken wire)

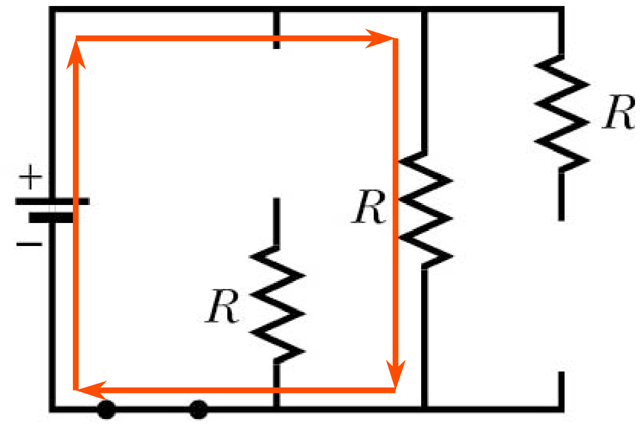
> Long time later $i = \mathcal{E}/R$ (acts like simple wire)

Problem 31-5

- Have a circuit with resistors and inductors
- What is the current through the battery just after closing the switch?



- Inductor oppose change in current through it
- Right after switch is closed, current through inductor is 0
- Inductor acts like broken wire



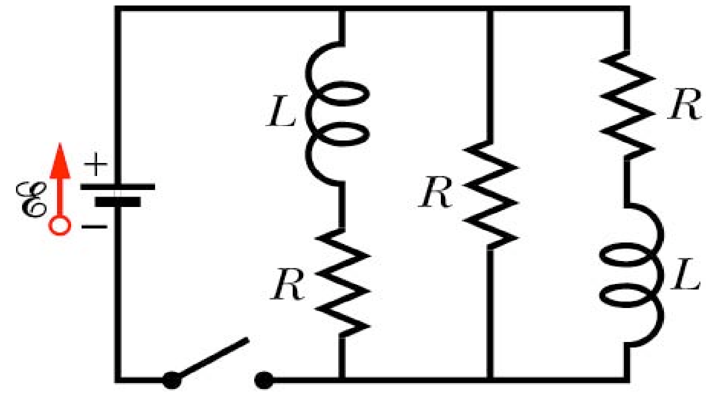
Problem 31-5, continued

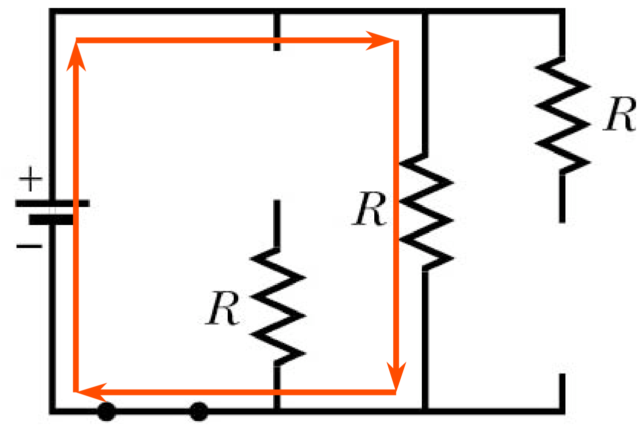
- Apply loop rule

$$\mathcal{E} - iR = 0$$

- Immediately after switch closed, current through the battery is

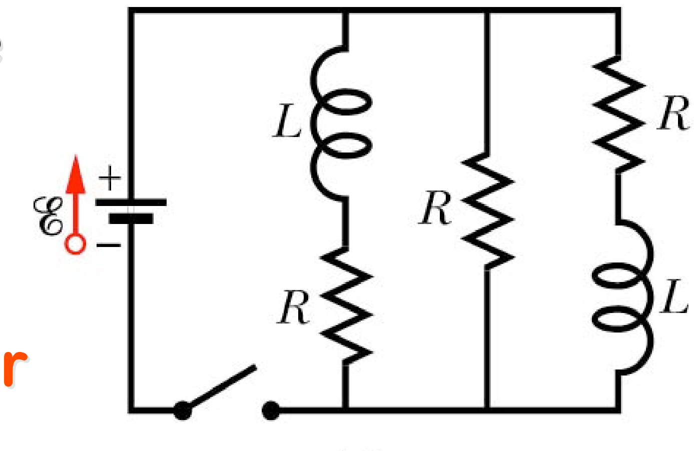
$$i = \frac{\mathcal{E}}{R}$$





Problem 31-5, continued

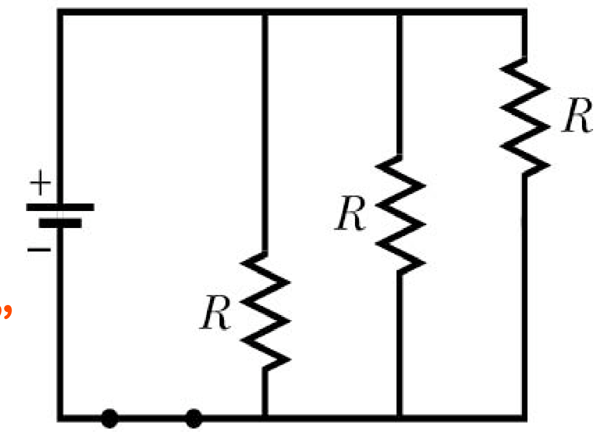
- What is the current through the battery a long time after the switch has been closed?
- Currents in circuit have reached equilibrium so inductor acts like simple wire
- Circuit is 3 resistors in parallel



$$i = \frac{\mathcal{E}}{R_{eq}}$$

$$R_{eq} = \frac{R}{3}$$

Remember the "water slides"



(c)

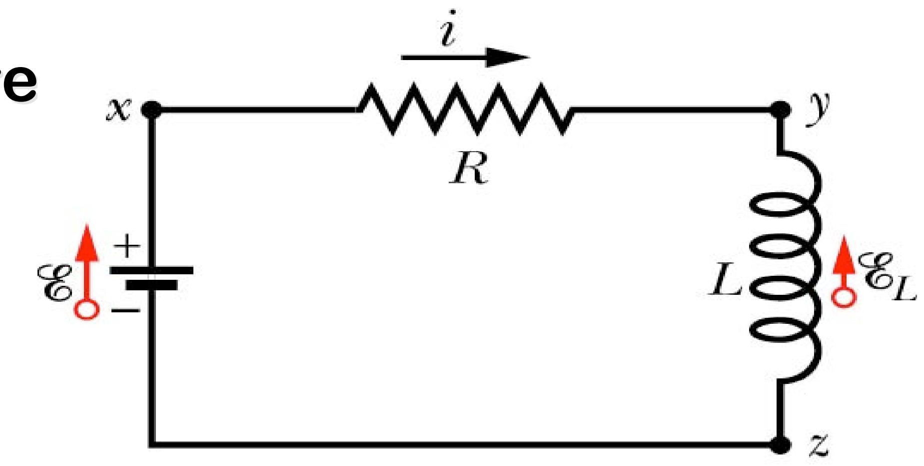
Energy

- How much energy is store in a *B* field?
- Conservation of energy expressed in loop rule

$$\mathcal{E} = L\frac{di}{dt} + iR$$

- Multiply each side by *i*

$$\mathcal{E}i = Li\frac{di}{dt} + i^2R$$



- $P=i\mathcal{E}$ is the rate at which the battery delivers energy to rest of circuit
- $P=i^2R$ is the rate at which energy appears as thermal energy in resistor

Energy (2)

- Middle term is rate at which energy dU_B/dt is stored in the B field

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

- Energy stored in magnetic field

$$U_B = \frac{1}{2}Li^2$$

- Similar to energy stored in electric field

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

Energy Density

- What is the energy density of a *B* field?
- Energy density, u_B is energy per unit volume

$$u_B = \frac{U_B}{Al}$$

- Magnetic energy density

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

- Similar to electric energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Where does this come from?

$$B = \mu_0 n i$$

$$L = l \mu_0 n^2 A \Longrightarrow$$

$$U_B = \tfrac{1}{2} i^2 L = \tfrac{1}{2} l \mu_0 n^2 i^2 A = \tfrac{1}{2} A l (n^2 i^2 \mu_0^2) / \mu_0$$

$$U_B / A l = \tfrac{1}{2} B^2 / \mu_0$$