

Fluctuations and Clustering in Heavy-Ion Collisions

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We propose a new theory to treat fluctuation phenomena in heavy-ion reactions. In practical terms, the method is an extension of the theories of the one-body density based on mean-field plus collisional dynamics. In an exploratory study of the $^{20}\text{Ne} + ^{20}\text{Ne}$ reaction, we find considerable fragmentation with a rapidly falling mass spectrum.

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There has been a large amount of theoretical work recently devoted to the study of the fragmentation of highly excited nuclei. The models discussed employ a broad variety of physical assumptions. They involve such concepts as thermodynamic phase transitions,¹ statistical equilibrium,² percolation,³ molecular dynamics,⁴ and more complicated hybrid models that use mean-field dynamics. By itself mean-field theory lacks enough fluctuation to produce realistic fragmentation. Up to now, the important fluctuations were introduced by *ad hoc* assumptions. In the work of Knoll and Strack⁵ the initial compressional phase of the reaction was assumed to produce a local equilibrium, and only the expansion phase was simulated by time-dependent Hartree-Fock (TDHF) theory. In another class of hybrid model,⁶ the Boltzmann-Uehling-Uhlenbeck (BUU) theory was used for the initial stage of the reaction. At some point in the time evolution, the single-particle density matrix is replaced by particle wave packets.

Here we shall present the beginnings of a theory that includes the important quantum effects and also has sufficient fluctuations to show qualitatively realistic fragmentation. Theories of nuclear dynamics are founded on various approximations to the wave function or propagator of the system. The TDHF theory follows when the wave function is approximated by a single Slater determinant. The extended TDHF theory,⁷ which is the quantum parent of the classical BUU theory, evolves the single-particle density, including the mixing of many Slater determinants perturbatively. However, the potential field is calculated from the single-particle density, and so does not have any additional fluctuations. As in the extended TDHF theory, we assume that the mixing of Slater determinants can be calculated perturbatively. We also assume that any coherence between the determinants can be ignored. The neglect of coherence is a crucial assumption which allows the evolution of the system to be calculated stochastically on a Markov chain, jumping from one configuration to another. Thus only

one determinant need be considered at a time, and the mean field will be correct for that state. The major ambiguity with this approach is the choice of single-particle basis for the determinants. The basis affects the evolution of the system if the coherence between determinants is neglected. Obviously, the basis should be chosen to minimize the error in this fundamental approximation. Couplings between configurations are minimized by our choosing orbitals that localize the particles as much as possible in coordinate space. Of course, the perturbative scheme for the mixing of determinants requires that the orbitals be chosen in a way to allow energy-conserving jumps from one determinant to another.

We are still far from realizing this theory at a quantum-mechanical level, but the essential ideas can be applied to a practical semiclassical model of the evolution of the system. Slater determinants are propagated by use of the Vlasov equation and the test-particle method.⁸ The jumping of the Slater determinants is treated by the Boltzmann collision integral, except that the collisions are treated as branching points in the evolution of the phase-space density. This is the fundamental difference from the conventional BUU approach in which the collision term is treated as a continuous source term. That method is presently implemented by representation of the single-particle distribution function by collections of test particles⁹ or by sums of Gaussian functions.¹⁰ In the method of Ref. 9, the test particles are grouped into parallel simulations, and the mean field is calculated from an ensemble average. In the limit of a large number of simulations, fluctuations due to nucleon-nucleon collisions are completely averaged out.

It is assumed in the conventional BUU approach that the ensemble average is a good approximation to the quantum-mechanical expectation value of a measurement,

$$\langle a \rangle = \sum_{\alpha} \alpha P_{\alpha}$$

This approach has been successful in the explanation of one-body observables such as proton spectra and collective flow effects.⁹ The BUU ensemble average cannot converge to $\langle a \rangle$, however, if channels become important which are beyond the scope of a one-body density description. The BUU theory fails to describe the formation of composite fragments because particle-particle correlations are important for the relevant observables.

To describe the Slater determinants we represent their single-particle Wigner distribution functions by test particles. We have

$$N = n(A_t + A_p) \quad (1)$$

test particles all together, where A_t and A_p are the masses of the target and projectile, and n is the number of test particles per nucleon. In the calculation below, $n=200$ was used. The phase-space distribution function $f(\mathbf{r}, \mathbf{p}, t=0)$ is then approximated by division of the occupied phase space into small cells Ω_i , of equal volume. In each cell we place randomly N_i test particles. The N_i are determined by

$$N_i = \lambda^{-1} \int_{\Omega_i} f(\mathbf{r}, \mathbf{p}, 0) d^3r d^3p, \quad (2)$$

where the normalization constant λ is

$$\lambda = N^{-1} \int_V f(\mathbf{r}, \mathbf{p}, 0) d^3r d^3p. \quad (3)$$

N is given by Eq. (1). V is the total occupied phase-space volume.

Given an initial phase-space distribution function $f(\mathbf{r}, \mathbf{p}, 0)$, Eq. (2) provides a prescription for the distribution of N test particles to approximate f . In principle one could start from a shell-model determinantal wave function and perform a Wigner transformation to get $f(\mathbf{r}, \mathbf{p}, 0)$. However, to be consistent with our classical propagation of the density, we choose to start from the distribution obtained from the Thomas-Fermi *Ansatz* in the rest frames of target and projectile,

$$f(\mathbf{r}, \mathbf{p}, 0) = \theta(p_F[\rho(r)/\rho_0]^{1/3} - p), \quad (4)$$

where $\rho(r)$ is the ground-state density distribution of the nucleus. The distributions are then boosted corresponding to the desired beam energy.

Similar to the BUU approach, the basic equation governing the time evolution of the system is

$$\begin{aligned} \partial_t f_1 + \mathbf{v} \cdot \nabla_r f_1 - \nabla_r U \cdot \nabla_p f_1 = & \frac{4}{(2\pi)^3} \int d^3p_2 d^3p_3 d\Omega v_{12} \frac{d\sigma}{d\Omega} \\ & \times \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) [f_1 f_2 (1 - f_3)(1 - f_4) - f_3 f_4 (1 - f_1)(1 - f_2)], \end{aligned} \quad (5)$$

with $f_j = f(\mathbf{r}_j, \mathbf{p}_j, t)$, v_{12} the relative velocity between test particles 1 and 2, and $d\sigma/d\Omega$ the (energy-dependent!) nucleon-nucleon cross section. For the mean-field potential we chose a density-dependent Skyrme parametrization,

$$U(\rho(\mathbf{r})) = -(124 \text{ MeV})\rho(\mathbf{r})/\rho_0 + (70 \text{ MeV})[\rho(\mathbf{r})/\rho_0]^2. \quad (6)$$

This potential reproduces nuclear-matter saturation properties and is known as the stiff equation of state. The coordinate-space density $\rho(\mathbf{r}, t)$ is obtained from the phase-space distribution $f(\mathbf{r}, \mathbf{p}, t)$ via

$$\rho(\mathbf{r}, t) = \int f(\mathbf{r}, \mathbf{p}, t) d^3p. \quad (7)$$

In BUU theory the right-hand side of Eq. (5) is treated as a continuous source term, since all test particles in every group are allowed to undergo collisions. This results in a gradual change in the overall momentum distribution of the system. We now wish to model the discontinuous change of Slater determinants coming from two-particle transitions. We therefore suppress the collisions of test particles by a factor $1/n$. If two test particles successfully collide, the momenta are changed not only of these two but also of $2(n-1)$ contiguous particles in phase space. This corresponds to collisions of two physical particles. The shape of the phase-space volume should maximize the spatial localization of the particles, consistent with overall energy conservation.

Our actual procedure for moving test particles is as follows. Suppose in a collision two test particles at coordinates $(\mathbf{r}_1, \mathbf{p}_1)$ and $(\mathbf{r}_2, \mathbf{p}_2)$ scatter with a momentum

transfer

$$\Delta \mathbf{p} = \mathbf{p}'_1 - \mathbf{p}_1 = -(\mathbf{p}'_2 - \mathbf{p}_2).$$

The phase-space volume is defined as a sphere of radius R_1 about \mathbf{r}_1 and of radius P_1 about \mathbf{p}_1 . The two radii are chosen so that the number of test particles contained in the volume is n . We have to specify one more condition to determine the relative magnitudes of R_1 and P_1 . Since R_1 and P_1 are limited by the radius R_N of the nucleus and the Fermi momentum p_F , we chose $R_1/P_1 = R_N/p_F$. Now all n test particles within the (R_1, P_1) sphere are given the same momentum change $\Delta \mathbf{p}$. In the same way we find radii R_2 and P_2 for the second particle and change the momentum of n test particles in its vicinity by $-\Delta \mathbf{p}$. This method ensures total momentum conservation. It approximately preserves the total phase-space volume occupied, namely $f^2 = f$ or 1 in the final state. This is required for the condition $f^2 = f$, the semi-classical equivalent to the quantum condition on a Slater determinant that $\rho^2 = \rho$.

Between collision times t_j and t_{j+1} the evolution of the

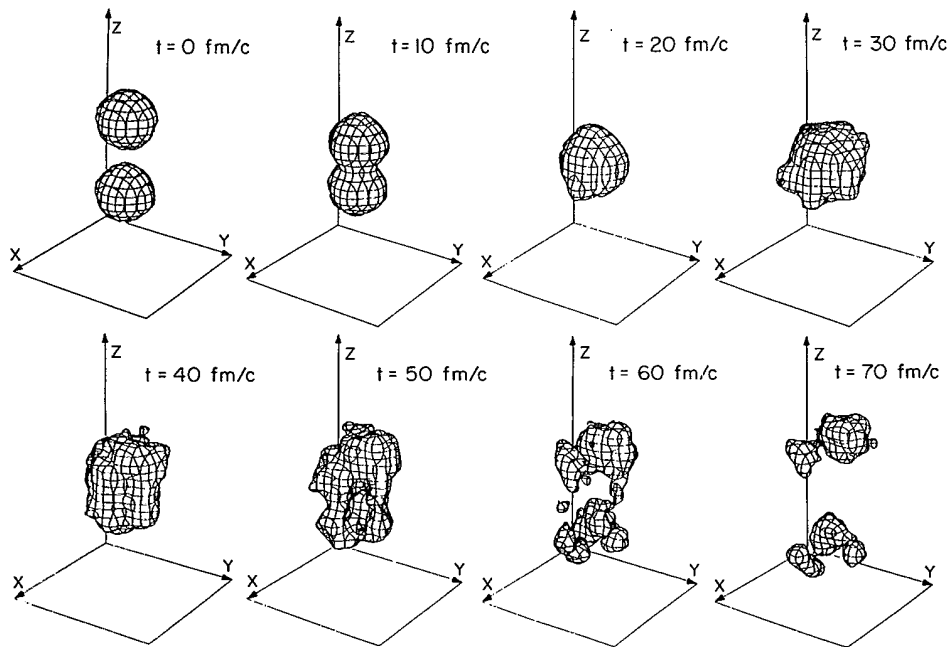


FIG. 1. Time evolution of a central $^{20}\text{Ne} + ^{20}\text{Ne}$ collision at a beam energy of 100 MeV/nucleon. Displayed is the density surface $\rho(x,y,z) = 0.02 \text{ fm}^{-3}$. Lines of constant x , constant y , and constant z are plotted on this surface.

system is governed by the solution of the Vlasov equation

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U \cdot \nabla_{\mathbf{p}} f = 0, \quad t_j \leq t \leq t_{j+1}. \quad (8)$$

This leads to equations of motion for the test particles,

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{E}, \quad \frac{d\mathbf{p}}{dt} = -\nabla_{\mathbf{r}} U(\rho(\mathbf{r})), \quad t_j \leq t \leq t_{j+1}, \quad (9)$$

which are solved by use of standard low-order Runge-Kutta techniques.

In Fig. 1 we have plotted the time evolution of a system of two ^{20}Ne nuclei evolving according to our model. We have chosen a beam energy of 100 MeV/nucleon in a head-on collision. At time intervals of $(10 \text{ fm})/c$ we display in a quasi three-dimensional graph the surface for which the nuclear density $\rho(x,y,z)$ exceeds a value of 0.02 fm^{-3} . Lines of constant x , constant y , and constant z are plotted on the surface. The total volume of the coordinate space, $20 \times 20 \times 30 \text{ fm}^3$, is indicated by the lengths of the coordinate axes.

At $t = 0 \text{ fm}/c$ both nuclei have spherical shapes and their surfaces are 2 fm apart. At $t = (10 \text{ fm})/c$ the initial penetration is visible. Collisions start to occur at roughly this time. By $t = (20 \text{ fm})/c$, the system has reached maximum overlap in coordinate space, and most of the nucleon-nucleon collisions are over by $t = (30 \text{ fm})/c$. In the example shown, eighteen collisions occur, thirteen of them by $t = (30 \text{ fm})/c$. At later times the system expands to lower densities. This is shown by the fact that the surface $\rho(x,y,z) = 0.02 \text{ fm}^{-3}$ takes up a larger

volume in coordinate space. The fluctuations induced by the nucleon-nucleon collisions begin to show up in the irregular shape of the density distribution. At $t = (60 \text{ fm})/c$ the density fluctuations are so large that the system breaks up into several fragments which are then moving away from each other.

We have calculated a mass-yield distribution using our model, for the 100-MeV/nucleon ^{20}Ne reaction. This is accomplished with the following algorithm. We use the final-state distribution to determine the density in a cubic array of coordinate-space cells, and mark the cells in which the density exceeds a value ρ_{cut} . We then determine which of these cells are connected, with a cluster search similar to the one used in percolation models. Summation of the density of all cells in a cluster yields

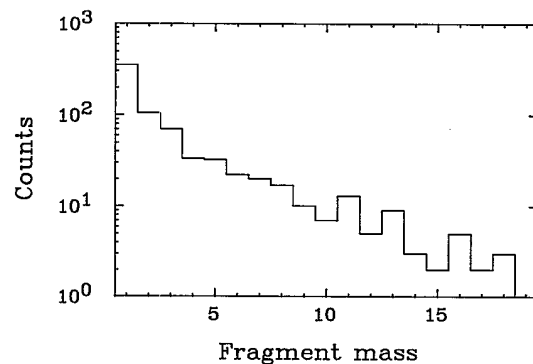


FIG. 2. Mass-yield curve for the $^{20}\text{Ne} + ^{20}\text{Ne}$ reaction at 100 MeV/nucleon. A sample of 100 central events was used.

the cluster mass.

In order for this method to have some physical significance the outcome of the cluster search should not be strongly dependent on ρ_{cut} , provided that it is chosen reasonably. As reasonable values for ρ_{cut} we consider the interval $\rho_0/10 < \rho_{\text{cut}} < \rho_0/4$. We have used different values of ρ_{cut} in this interval and found only negligible dependence of the fragment yields on its value.

In Fig. 2 we have displayed the mass yield resulting from a simulation of 100 central events for the above reaction. The mass yield is decreasing rapidly as a function of fragment mass. The dependence is about $A^{-1.6}$ in a power-law fit. However, at the present exploratory stage of our calculations, we do not feel we can quantitatively compare our results with experiment.

We have presented a model which is obviously superior to the BUU theory, since it does not utilize an ensemble-averaged mean field and contains fluctuations based on a definite quantum approximation scheme. It is of interest now to see whether these fluctuations are adequate to describe the data, or whether less restrictive assumptions must be used in making a dynamic theory.

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