# Mass Dependence of the Disappearance of Flow in Nuclear Collisions

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The disappearance of collective flow in nucleus-nucleus collisions occurs at an incident energy ($E_{\text{bal}}$) where the attractive scattering dominant at low energies balances the repulsive scattering dominant at high energies. We have performed the first systematic study of the entrance-channel mass dependence of the disappearance of flow and hence $E_{\text{bal}}$. The new data presented for the C+C, Ne+Al, Ar+Sc, and Kr+Nb systems show that $E_{\text{bal}}$ scales as $A^{-1/3}$ where $A$ is the mass of the combined system. Boltzmann-Uehling-Uehlenbeck model calculations show trends which are in qualitative agreement with these new results.

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The study of global collective variables in nucleus-nucleus collisions can provide information about the nuclear equation of state (EOS) [1–3]. In recent years much emphasis has been placed on the study of collective flow and several groups have performed experiments to study its disappearance [4–9]. The disappearance of collective flow is predicted to occur at an incident energy (termed the balance energy, $E_{\text{bal}}$ [4]) corresponding to the point where the attractive scattering, dominant at incident energies around 10 MeV/nucleon, balances the repulsive interactions observed at energies around 400 MeV/nucleon [4,10–14]. Attractive scattering corresponds to negative scattering angles which can be related to partial orbiting of the two nuclei. Repulsive scattering corresponds to positive angle scattering and can be visualized as the two nuclei bouncing off each other. Because the magnitude of parameters of the EOS can be related to the dominance of repulsive or attractive scattering, there is great value in the determination of $E_{\text{bal}}$. In this Letter we report results from the first systematic study of the disappearance of flow in nucleus-nucleus collisions. We present new data for the C+C, Ne+Al, Ar+Sc, and Kr+Nb systems which show that $E_{\text{bal}}$ scales as $A^{-1/3}$, where $A$ is the mass of the combined projectile-target system. The general trend of this result is reproduced by a microscopic transport theory in the form of Boltzmann-Uehling-Uehlenbeck (BUU) model calculations [15] confirming the interpretation of $E_{\text{bal}}$. This dependence on $A^{-1/3}$ has been noted also in Ref. [13]. We interpret the scaling of $E_{\text{bal}}$ with mass to be the result of a competition between the dominantly attractive mean field interaction which scales as the surface area, $A^{2/3}$, and the dominantly repulsive nucleon-nucleon scattering which scales as the volume of the two nuclei, $A$ [16].

The present measurements were carried out with the Michigan State University $4\pi$ Array [17] at the National Superconducting Cyclotron Laboratory (NSCL) using beams from the K1200 cyclotron. The $4\pi$ Array consists of a main ball of 170 phoswich counters (arranged in 20 hexagonal and 10 pentagonal subarrays) covering angles from 23° to 157° and a forward array of 45 phoswich counters covering angles from 7° to 18°. The 30 Bragg curve counters installed in front of the hexagonal and pentagonal subarrays were operated in ion chamber mode (in three different experiments) with pressures of 500, 250, and 100 torr of P5 (95% argon, 5% methane), $CF_4$, and $C_2F_6$ gases, respectively. The hexagonal anodes of the five most forward Bragg curve counters are segmented, creating a total of 55 separate $\Delta E$ gas counters.

The Bragg curve counters served as $\Delta E$ detectors for charged particles that stopped in the fast plastic scintillator of the main ball. Consequently, the Array was capable of detecting charged fragments from $Z = 1$ to $Z = 12$. Low energy thresholds (in each experiment) were ≈17 MeV/nucleon, ≈3 MeV/nucleon, and ≈5 MeV/nucleon for fragments of $Z = 1$, 3, and 12, respectively. The targets used in the experiments consisted of 1 mg/cm$^2$ natural carbon, 1 mg/cm$^2$ natural aluminum, 1.6 mg/cm$^2$ natural scandium, and 1 mg/cm$^2$ natural niobium. Beam intensities were approximately 100 particle pA. The beam energies used were 55 to


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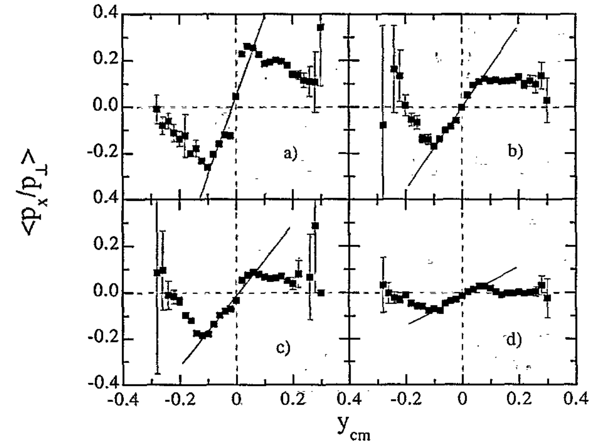


FIG. 1. Average transverse momentum in the reaction plane divided by the total transverse momentum versus the center of mass rapidity for He fragments from (a) 55 MeV/nucleon C+C, (b) 75 MeV/nucleon Ne+Al, (c) 45 MeV/nucleon Ar+Sc, and (d) 35 MeV/nucleon Kr+Nb. The solid lines are fitted over the region of $0.8y_t \leq y_{c.m.} \leq 0.8y_p$ where $y_t$ and $y_p$ are the target and projectile rapidities, respectively, in the center of mass.

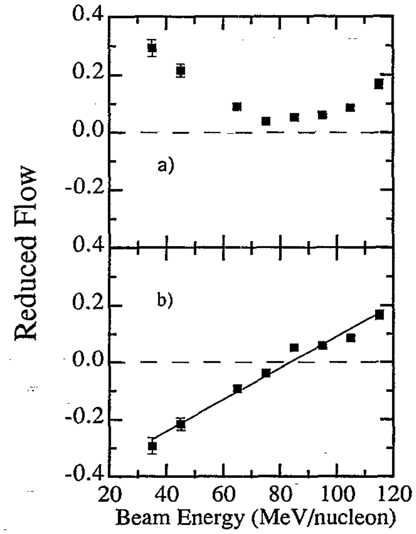


FIG. 2. Reduced flow versus incident energy for protons from Ar+Sc. (a) Reduced flow values as extracted. (b) Inverted reduced flow values for incident energies below 85 MeV/nucleon. Solid line is a linear fit.

155 MeV/nucleon for $^{12}$C, 55 to 140 MeV/nucleon for $^{20}$Ne, 35 to 115 MeV/nucleon for $^{40}$Ar, and 35 to 75 MeV/nucleon for $^{86}$Kr. Data were taken with a minimum bias trigger (charged particle multiplicity $m \geq 2$) and a more central trigger ($m \geq 5$).

To extract collective flow in nucleus-nucleus collisions one must determine the impact parameter, $b$, and the reaction plane. We determined the centrality of each event by way of cuts on the total transverse momentum. Central collisions corresponding to an average impact parameter $\approx 0.40b_{\max}$ were used for each system. Here, $b_{\max}$ is the maximum estimated impact parameter. The reaction plane was determined using the method of azimuthal correlations [18].

Having selected central collisions and determined the reaction plane for each event, the average fraction of the total transverse momentum in the reaction plane, $\langle p_x/p_\perp\rangle$, was evaluated and plotted as a function of the center of mass rapidity, $y_{c.m.}$. Figure 1 shows a selected set of these plots ($\langle p_x/p_\perp\rangle$ vs $y_{c.m.}$) for He fragments from collisions of 55 MeV/nucleon C+C (a), 75 MeV/nucleon Ne+Al (b), 45 MeV/nucleon Ar+Sc (c), and 35 MeV/nucleon Kr+Nb (d). The characteristic shape commonly associated with flow is quite evident in these figures. The errors shown in Figs. 1(a)–1(d) are statistical, and the solid lines correspond to linear least square fits for the region $0.8y_t \leq y_{c.m.} \leq 0.8y_p$ where $y_t$ and $y_p$ are the target and projectile rapidities, respectively, in the center of mass. The slope of each line is the reduced flow.

The reduced flow is shown as a function of beam energy for protons from collisions of Ar+Sc in Fig. 2. The top panel of Fig. 2(a) clearly shows that the reduced flow goes through a minimum. Because our measurements are unable to distinguish the sign (+ or −) of collective flow, such a minimum is indicative of the disappearance of flow. In order to facilitate the determination of $E_{\rm bal}$, the extracted values of the reduced flow for beam energies below 85 MeV/nucleon are plotted as negative values in Fig. 2(b). This procedure is in accordance with expectations from BUU calculations. $E_{\rm bal}$ is determined from this plot [Fig. 2(b)] by performing a linear least square fit to the reduced flow values. The solid line shown in Fig. 2(b) represents such a fit. The value of $E_{\rm bal}$ is fixed by the point or intersection of this line with the dashed line (zero reduced flow). In a similar manner, $E_{\rm bal}$ was extracted for C+C, Ne+Al, and Kr+Nb. Varying the point at which the values for the reduced flow change sign makes little difference in the resulting value for $E_{\rm bal}$. Nevertheless, this effect is included in the estimated errors for $E_{\rm bal}$.

The fact that $E_{\rm bal}$ is nearly independent of particle type is demonstrated in Fig. 3 for the Ar+Sc system. Here, we plot the reduced flow for $p$, $d$, $t$, and He particles as a function of incident energy. The solid lines represent linear fits to the reduced flow values of $p$, $d$, $t$, and He particles, respectively. All the values for reduced flow have been taken as negative below 85 MeV/nucleon. These lines intersect the zero reduced flow line at approximately the same point indicating similar $E_{\rm bal}$ for each particle.

The $E_{\rm bal}$ values extracted for the four systems are summarized in Table I. The value listed for each system represents an average of the $E_{\rm bal}$ values obtained for each particle ($p$, $d$, $t$, and He). The quoted error bars include

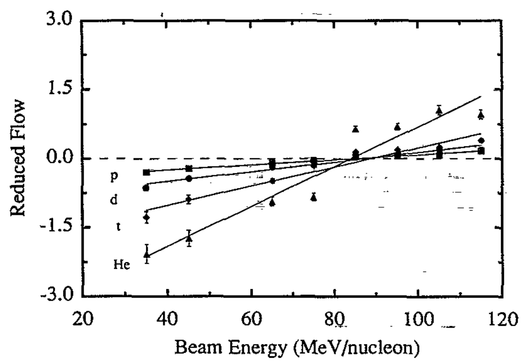


FIG. 3. Reduced flow versus incident energy for $p$, $d$, $t$, and He fragments from collisions of Ar+Sc. The solid lines are linear fits. Negative flow values are assigned as described in the text.

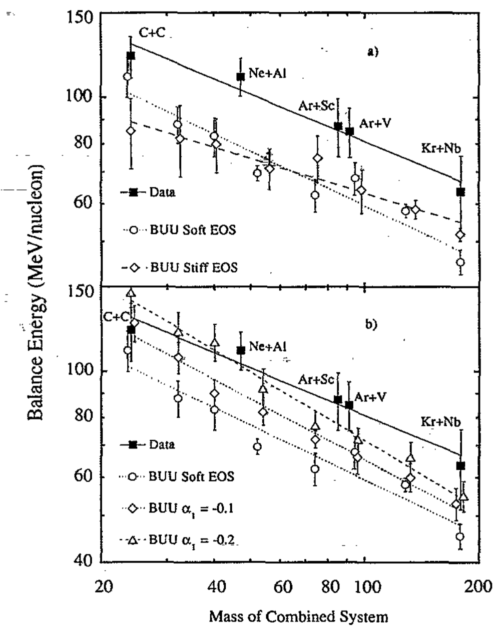


FIG. 4. (a) Measured values of the balance energy for C+C, Ne+Al, Ar+Sc, and Kr+Nb compared with the predictions of the BUU model for a soft and a stiff equation of state. The lines correspond to power law fits. (b) Measured values of the balance energy for C+C, Ne+Al, Ar+Sc, and Kr+Nb compared with the predictions of the BUU model for a soft equation with a density dependent reduction of the in-medium cross sections of 0%, 10%, and 20% (see text). The experimental value for Ar+V is from Ref. [7].

both statistical and systematic errors. We have also plotted $E_{\rm bal}$ as a function of the mass of the entrance channel in Fig. 4. The solid lines in this figure correspond to a fit of the form $A^{-1/3}$, where $A$ is the mass of the combined system. The results of BUU model calculations [15] are also included in Fig. 4(a) for both a soft ($K = 200$ MeV) EOS and a stiff ($K = 380$ MeV) EOS. The lines correspond to a fit of the form $A^{-\tau}$. For the soft EOS $\tau = 0.32 \pm 0.03$ and for the stiff EOS $\tau = 0.28 \pm 0.05$. Thus the BUU calculations show little dependence on $K$. We have previously demonstrated that the BUU predictions for $E_{\rm bal}$ are independent of the acceptance of the apparatus [4].

Shown in Fig. 4(b) are BUU calculations with $K = 200$ MeV in which the in-medium cross sections are reduced. In previous studies the free nucleon-nucleon cross section was multiplied with an overall constant scaling factor [4,12,15]. However, this approach fails when one has collisions in low-density nuclear matter where the in-medium cross section should approach its free-space value. A more realistic approach uses a Taylor expansion of the in-medium cross section in the density variable. We introduce the dimensionless parameter $\alpha_1$ to describe the in-medium cross sections in terms of the free cross sections, $\sigma_{NN}^{\text{in-medium}} = (1 + \alpha_1 \frac{\rho}{\rho_0})\sigma_{NN}^{\text{free}}$. In principle, $\alpha_1$ is dependent on energy, but we have here, as a first approximation, taken $\alpha_1$ as energy independent. The BUU calculations with $\alpha_1 = -0.1$ predict a higher $E_{\rm bal}$ than the calculation incorporating the free cross section. The calculations with $\alpha_1 = -0.2$ roughly reproduce the observed values for $E_{\rm bal}$ as can be seen in Fig. 4(b). The dotted lines correspond to power law fits. The BUU calculations with different $\alpha_1$ clearly have a different slope in Fig. 4(b), none of which completely reproduce the quantitative behavior of $E_{\rm bal}$ although the qualitative behavior is correct. Clearly the BUU calculations with a soft EOS and $\alpha_1 = -0.2$ most closely reproduce the measured values of $E_{\rm bal}$.

The dependence of $E_{\rm bal}$ on the mass of the combined system can be interpreted as a signature of the competition between the attractive mean field and the repulsive nucleon-nucleon interactions. The mean field can be associated with the surface of the two interacting nuclei and hence should scale as $A^{2/3}$. Similarly, the repulsive nucleon-nucleon interaction should scale as $A$. This being the case, one expects $E_{\rm bal}$ to show the mass dependence illustrated in Fig. 4. The fact that BUU calculations reproduce this dependence qualitatively serves to confirm the interpretation of the disappearance of flow as a balancing of the attractive scattering (dominant at low energies) by the repulsive nucleon-nucleon scattering (dominant at high energies). However, the BUU calculations cannot completely reproduce the dependence of

TABLE I. Extracted values for the balance energy.

| System | Balance energy (MeV/nucleon) |
|---|---|
| C+C | 122±12 |
| Ne+Al | 111±10 |
| Ar+Sc | 87±12 |
| Kr+Nb | 64±12 |

$E_{bal}$ on the size of the system.

In conclusion, we have measured $E_{bal}$ for C+C, Ne+Al, Ar+Sc, and Kr+Nb. These values for $E_{bal}$ scale as $A^{-1/3}$. The observed balance energies do not depend on the particle type. BUU calculations reproduce the scaling with mass but underpredict $E_{bal}$. A density dependent reduction in the in-medium nucleon-nucleon cross sections corresponding to 20% at normal nuclear density qualitatively reproduces the mass dependence of $E_{bal}$. However, the BUU calculations show little dependence on the stiffness of the EOS. These results confirm that the disappearance of flow is related to the balance between the attractive and repulsive components of the nuclear equation of state.

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