Worksheet #2 - PHY102 (Spr. 2008)

Formats, Palettes and List operations (Vectors)

Formating

As you have probably read in the the introduction notebook, *Mathematica* has numerous formatting options, which are found under Format→Style. There are 16 different options, with many being text options. *Input* is the default and is the one which must be used when carrying out mathematical operations, plotting graphs etc. At the top of each worksheet use a cell with *Title* format to enter your name and the worksheet number. It is also a good idea to separate problems using horizontal lines, which can be found under *Insert → Horizontal Lines.*

Palettes

Last week we did derivatives and integrals using the full *Mathematica* commands. Many of these commands may also be entered using *Palettes*, which are accessed using the Palette button at the top of the worksheet. *Basic Math Input* is a good one for many worksheets and you may have several of them open at one time.

When you get into trouble

Sometimes you will try to use a variable in more than one way. This can confuse Mathematica. There are several ways to clear a variable a, for example

$a = .$ which clears a numerical assignment to a, and

$Clear[a[x]]$ which clears a function assignment to $a[x]$.

If you want to remove all of your prior definitions you can use,

$Remove[\text{“}Global\text{‘} * \text{”}]$

At some point Mathematica will get really unhappy and start doing a really long winded calculation which you did not think you asked it to do. In that case you can go to the *Evaluation* tab, and click on *Abort evaluation.* Sometimes that does not work, in which case you can click on *Quit kernel.* This stops the mathematica kernel and you loose the evaluations you have already carried out in the whole worksheet. If neither of these works, you can try screaming and ranting about stupid computer programs, which may get the TA's attention.

Lists and Vectors

By now you must have read about vectors. A vector is a quantity which, unlike a scalar, can have many components. For example in Newton's second law of motion

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} \tag{1}$$

the quantity m (mass) is a scalar. But the force $\vec{F}$ and the acceleration $\vec{a} = \frac{d^2\vec{r}}{dt^2}$ are vectors. As you can see in Eq. (1), and which is true in general, multiplying a vector $\vec{a}$ with a scalar m, gives a vector $\vec{F}$. A vector is described by its components in a chosen co-ordinate system. For example a vector $\vec{A}$ in cartesian co-ordinates is given by, $\vec{A} = (A_x, A_y, A_z)$.

In Mathematica vectors are represented in the same way. In Mathematica this object is called a *list*, because it can be used for more general objects such as matrices and tensors. In this worksheet we just work with vectors.

Type "A = {Ax, Ay, Az} " (ie just type the quantity in quotations). This means Mathematica associates the object A with the list {Ax, Ay, Az}. Now type "B = {Bx, By, Bz}". Type "Dot[A,B]". This will give the dot product $\vec{A} \cdot \vec{B}$=AxBx+AyBy+AzBz which is the same as $|A||B|cos(\theta)$, where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.

Likewise, the cross product of two vectors ($\vec{A} \times \vec{B}$) yields another vector $\vec{C}$ = {AyBz-AzBy, AzBx-AxBz, AxBy-AyBx}. Type "Cross[A,B]" and verify that you indeed get the above expression in terms of the components of $\vec{A}$ and $\vec{B}$. Unit vectors can be easily written with lists as: $\hat{x}$ = {1,0,0}, $\hat{y}$ = {0,1,0}, $\hat{z}$ = {0,0,1}. Check with Mathematica that $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$.

You can see that the elements in the list {Ax, Ay, Az} of the vector $\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ are its x, y, and z components. How do we access the individual components from A?.

Type "A[[2]]" and check that this gives Ay. How would you get *Mathematica* to print out the second element of the cross product "Cross[A,B]"?

Assignment 2. - Hand in by 6pm Friday Jan. 25th

Problem 1. Consider two vectors $\vec{A} = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$, and $\vec{B} = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$. Using *Mathematica*:
(i) Check that they are both of unit magnitude.
(ii) Find $\vec{A} \cdot \vec{B}$.
(iii) Find the angle between these two vectors.
(iv) Find the cross product of these two vectors.

Problem 2. Consider the unit vectors along x, y, and z directions: $\hat{x} = \{1,0,0\}$ $\hat{y} = \{0,1,0\}$ $\hat{z} = \{0,0,1\}$. Verify: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{y} \times \hat{z} = \hat{x}$, $\hat{z} \times \hat{x} = \hat{y}$.

Problem 3. Verify that for any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

Problem 4. The motion of a particle is given by $\vec{r}(t)$=a($\hat{x}$cos(ωt) + $\hat{y}$sin(ωt)) Find its velocity $\vec{v}$. Calculate $\vec{\Omega} \times \vec{r}$, where $\vec{\Omega} = (0, 0, \omega)$, and verify that $\vec{v} = \vec{\Omega} \times \vec{r}$. Do you recognise this motion? Plot the motion to confirm your intuition (use the help menu to look up how to use the command *ParametricPlot* for this problem - you will need to choose values for a and ω).