

# PHY481 - Lecture 1

## Pollack and Stump (PS): Chapter 1 and Sections 2.1.1 and 2.1.2

### A. A Brief History

In early history the special magnetic and electric properties of some materials were noted. The lodestone is a magnetic rock while amber was considered special due to its ability to produce static discharge.

Scientific ideas about charge and the flow of charge were developed by Gray, Franklin etc in the middle of the 18th century. Coulomb made these ideas concrete by measuring the force between charges and developing a formula to describe these forces (1780's). This formula is the basis of electrostatics (stationary charge distributions). However it is often easier to use a reformulation of this law to a form called Gauss's law to solve problems.

Ampere and Oersted noticed the fact that a DC current generates a magnetic field. Biot and Savart developed a formula to describe this. This occurred in the 1820's. Analysis of the magnetic fields generated by DC currents is called magnetostatics. The Biot-Savart law, or Ampere's law is the basis of magnetostatics. Ampere also noticed that a time varying electric field also induces a magnetic field. Ampere's law describes this phenomenon (1820's).

Faraday observed that a time varying magnetic flux leads to an induced emf (1831). Joseph Henry also observed indications of this in 1830, but Faraday's experiments are considered definitive. Faraday's law describes these phenomenon and is the basis of electric motors and generators.

Heaviside and Gibbs wrote down the equations of electrostatics, magnetostatics, magnetic induction and a generalized form of Ampere's law in a unified form building upon two of Maxwell's papers from (1861-65). In the early literature the collection of these four equations, now known as Maxwell's equations, were therefore called the Hertz-Heaviside equations. Hertz was included because he did experiments verifying Maxwell's prediction of EM waves. However Einstein decided to call them the Maxwell-Hertz equations and the name stuck. Anyway they are considered a complete theory of classical electricity and magnetism or electromagnetism and can be considered the basis of this course.

Often an additional equation is added to this set - the Lorentz Force law. The Lorentz force law describes the way in which electric and magnetic fields effect a moving charge. This equation was developed by Lorentz (1892/1895).

The chronology of these discoveries and other physics discoveries is available at the history of physics from the aps

### **B. Basic concepts about charge**

- There are two types of charge, positive and negative.
- Charge is conserved.
- Charge is quantized.  $e = 1.6 \times 10^{-19}C$ .

### **C. Basic concepts about materials**

- Conductor - If there is a voltage across a conductor, current flows.
- Insulator - Even if there is a voltage across an insulator, current does not flow.
- Semiconductor = Insulator at low voltage and temperature.
- Superconductor = Conductor, for static charges and no applied magnetic field.

### **D. The first quantitative EM law: Coulomb's law (1780's)**

The starting point in electrostatics is Coulomb's law, which gives the force between two stationary charges,

$$\vec{F} = k \frac{Qq}{r^2} \hat{r} = k \frac{Qq}{r^3} \vec{r} \quad (1)$$

- $Q, q$  are stationary charges. Their units are coulombs (C)
- $\hat{r}$  is a unit vector along the line between the two charges.
- $\vec{r}$  is the vector distance between the two charges.
- $k = 9 \times 10^9 Nm^2/C^2 = 9 \times 10^9 kgm^3/C^2s^2$ .
- $k = 1/4\pi\epsilon_0$ .  $\epsilon_0$  is the permittivity of free space.

### **E. Gravitational force law : Measured by Cavandish 1790's**

$$\vec{F}_G = -G \frac{Mm}{r^2} \hat{r} \quad (2)$$

- $G = 6.67 \times 10^{-11} Nm^2/kg^2$ . The gravitational force is much weaker than the electrostatic force.

*Example*

Find the ratio of the magnitudes of the gravitational and electrostatic forces between two protons.

*Solution* The ratio of the magnitudes of the gravitational and electrostatic forces is,

$$\frac{F_G}{F} = \frac{GMm}{kQq} \quad (3)$$

For two protons, we have,  $M = m = 1.67 \times 10^{-27} \text{kg}$  and  $Q = q = 1.6 \times 10^{-19} \text{C}$ . Plugging these numbers yields,

$$\frac{F_G}{F}(\text{two protons}) = \frac{6.67 \times 10^{-11}(1.67 \times 10^{-27})^2}{9 \times 10^9(1.6 \times 10^{-19})^2} = 8.1 \times 10^{-35} \quad (4)$$

## F. Most charge is “bound”

Due to the fact that the force between charges is so large, most charge is bound. That means that negative and positive charges are close to each other in regions where the total charge is almost zero. This raises the question. Why don't charges self destruct by crashing into each other and annihilating? The answer is in quantum mechanics, which provides an understanding of how atoms are stable. Atoms are composed of electrons orbiting the nuclei which contains the protons. This provides a stable configuration of bound charges. Within electrostatics bound charges still form dipoles. There are many more charge dipoles than there are free charges. Many electrostatic effects are due to dipoles rather than free charges, so we need to understand dipoles well.

No free magnetic charges have been observed, so that N and S magnetic poles always come in pairs. Nevertheless the search of free magnetic charge, ie magnetic monopoles, continues. The sources of magnetic fields are currents or moving charges.

## G. Force between many charges - superposition

Force on a charge  $q$  due to many other charges,  $Q_1, Q_2, \dots, Q_n$  is just the sum of the forces due to each of these charges, ie.

$$\vec{F}_{tot} = \sum_{i=1}^n k \frac{Q_i q}{r_i^2} \hat{r}_i \quad (5)$$

This looks simple, but of course it is a vector sum, so the math can get messy.  $r_i$  is the distance between charge  $i$  and the charge  $q$ .  $\hat{r}_i$  is a unit vector along the vector which goes from charge  $q$  to charge  $Q_i$ .

The principle of superposition also applies when there is a continuous distribution of charge. For example charge distributions on rods, discs, spheres etc. However when treating these distributions, the sum in Eq. (5) becomes an integral. In treating these problems, we define a small element of charge  $dQ$ . This is the amount of charge in a small part of the continuous charge distribution. We shall consider three cases:

- Lines: Then  $dQ = \lambda dx$ , where  $\lambda$  is the linear charge density.
- Surfaces: Then  $dQ = \sigma dA$ , where  $\sigma$  is the surface charge density.
- Volumes: Then  $dQ = \rho dV$ , where  $\rho$  is the volume charge density.

## H. Maxwell's equations (Chapter 11 of PS) and a course outline

### 1. Gauss' law for electric field - the basis of electrostatics (Chapters 3-5 of PS)

The Electric flux,  $\phi_E$ , through a closed surface,  $A$  is proportional to the net charge,  $q$ , enclosed within that surface.

$$\phi_E = \oint \vec{E} \cdot \hat{n} dA = \frac{q}{\epsilon_0} \quad (6)$$

Differential form,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (7)$$

### 2. Gauss's law for magnetic field

The Magnetic flux,  $\phi_B$ , through a closed surface,  $A$  is equal to zero.

$$\phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \quad (8)$$

Differential form

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (9)$$

### 3. Generalized Ampere's law - Magnetostatics (Chapter 8 of PS)

The path integral of the magnetic field around any closed loop, is proportional to the current enclosed by the loop plus the displacement current enclosed by the loop.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_E}{dt} \quad (10)$$

Differential form

$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (11)$$

### 4. Faraday's law - magnetic induction (Chapter 10 of PS)

The emf induced in a closed loop, is proportional to the negative of the rate of change of the magnetic flux,  $\phi_B$ , through the closed loop,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} \quad (12)$$

Differential form

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (13)$$

## 5. Lorentz force law

The force on a charge moving with velocity  $\vec{v}$  in an electromagnetic field is given by,

$$\vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B}) \quad (14)$$

## 6. Also we will cover some parts of Electromagnetism and Matter

(Chapters 6,7,9 of PS).

## I. Vectors properties and vector operations

1. Consider vectors defined in a orthogonal basis set, with unit vectors  $\hat{i}, \hat{j}, \hat{k}$  in the x,y and z directions respectively. Then a vector  $\vec{A}$  may be written,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = (A_x, A_y, A_z) \quad (15)$$

2. Vector addition and subtraction is carried out component by component, for example for vectors  $\vec{A}$  and  $\vec{B}$  we have,

$$\vec{A} + \vec{B} = (A_x + B_x, A_y + B_y, A_z + B_z) \quad (16)$$

Pictorially this corresponds adding vectors “head to tail”.

3. Scalar multiplication is also carried out componentwise, so that,

$$c\vec{A} = (cA_x, cA_y, cA_z) \quad (17)$$

4. The length of a vector is,

$$A = |\vec{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2} \quad (18)$$

5 The dot product of two vectors is a scalar given by,

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos(\theta) \quad (19)$$

Note that the length of a vector  $\vec{A}$  is given by  $A = (\vec{A} \cdot \vec{A})^{1/2}$

6. The cross product of two vectors gives a third vector which is perpendicular to the plane of the two starting vectors. This is a very important property as often we want to use the normal to a plane, for example in defining the normal to a surface. The cross product is given by,

$$\vec{A} \wedge \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = AB \sin(\theta) \hat{n} = (A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x) \quad (20)$$