PHY481 - Lecture 28

Section 10.3-10.5 of PS, Sections 7.2.3, 7.2.4 of Griffiths

A. Faraday's law for many loops

Faraday's law is defined for one one, that is, the emf generated around a closed loops is, $\epsilon = -d\phi_B/dt$. However if we coil a wire so that the flux ϕ_B passes through many loops of the wire, then the total emf is multiplied by N, the number of loops in the wire. This is very important in a variety of applications ranging from magnetic field detection to transformers and antenna. If the flux through each of N loops is the same, then we have,

$$\epsilon = -N\frac{d\phi_B}{dt} \quad \text{for N loops} \tag{1}$$

B. Self Inductance

Faraday's law relates the induced emf to the change in magnetic flux, however the magnetic flux in circuits is often generated by currents, so we would like to relate flux to the current and to write Faraday's law in terms of the current. This is easy to do as the flux comes from the magnetic field and the magnetic field is proportional to the current, as is seen from the Biot-Savant law or from Ampere's law, therefore $\phi \propto i$. Using this proportionality we write Faraday's law as,

$$\epsilon = -N\frac{d\phi_B}{dt} = -L\frac{di}{dt} \tag{2}$$

This relation defines the self-inductance, L that is dependent on the geometry and on the materials used but not on the current or the voltage. Sometimes this relation is stated a different way, through the flux linkage $\lambda = Li = N\phi_B$, which leads to the same result as above.

Calculation of L is important as every circuit has a self inductance and its influence can be dominant in some frequency ranges. In general inductance is important at high frequency as there the current is changing rapidly so that the flux changes rapidly leading to high voltage through Faraday's law. In most cases calculation of L has to be carried out numerically or at least is quite complicated. However there are a few cases where it can be done analytically, and these cases are tabulated. We now go through three simple cases.

Solenoid

We find L by considering a solenoid with N turns and length l, and cross-sectional area A in air. A current i passes through the solenoid, and from Faraday's law we have $N\phi_B = Li$

(setting the constant of integration to zero). Since the magnetic field in a solenoid is constant and takes the value $B = ni\mu_0$ (where $n = N/l$), the flux is $\phi_B = NiA\mu_0/l$, so that,

$$N\phi_B = Li \quad \text{implies} \quad L = N^2 A\mu_0/l \tag{3}$$

Coaxial

Consider a co-axial consisting of a thin conducting cylinder of length l and of radius a that is concentric with a thin conducting cylinder of radius $b > a$. The flux through a loop that passes along the inner cylinder and then returns along the outer cylinder is given by,

$$\phi_B = \int \vec{B} \cdot d\vec{A} = l \int_a^b \frac{\mu_0 i}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} ln(b/a) \tag{4}$$

so the inductance is $L = \mu_0 l ln(b/a)/(2\pi)$.

Rectangular loop

It is harder to do a square loop as it is not trivial to find the magnetic field everywhere (we only found it on the z-axis). However if we take a rectangular loop with sides l and a and $l/a \to \infty$, then we can use the result for a field near a wire (multiplied by two), ie,

$$\phi_B = 2l \int_w^a \frac{\mu_0 i}{2\pi r} dr = 2\frac{\mu_0 ln(a/w) li}{2\pi} \tag{5}$$

The factor of two is due to the presence of two wires. The inductance is then, $L = \mu_0 ln(a/w) l/\pi$. In this expression, the wire is assumed to have radius w and current is assumed to be flowing at the surface of the wire.