PHY481 - Lecture 30

Chapter 9 of PS, Sections 7.2.3, 7.2.4 of Griffiths

A. Linear magnetic materials

Linear magnetic materials are characterized by a linear relation between the magnetization and the magnetic field intensity, $\vec{M} = \chi_m \vec{H}$, which is similar to the definition of linear dielectrics, $\vec{P} = \epsilon_0 \chi_e \vec{E}$ but not completely analogous. We also have,

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 (1 + \chi_m) \vec{H} = \mu \vec{H} \tag{1}$$

which is analogous to $\vec{D} = \epsilon \vec{E}$ in dielectrics. μ is the permeability. This should not be confused with the magnetic moment for a current ring, which is also sometimes called $\vec{\mu}$. This is horrible notation, but it is entrenched in the area. We shall have to live with it. Most of the time I use $\vec{m}$ for the magnetic moment and $\vec{M}$ for the magnetization.

B. Paramagnets, linear materials with $\chi_m > 0$

Paramagnets do not exhibit spontaneous magnetic order, nevertheless they can have large magnetic susceptibilities. The magnetic moment of paramagnetic materials tries to align in the direction of the applied magnetic field. Actually all materials will magnetically order at sufficiently low temperatures, but when the ordering temperature is very low, materials are called paramagnetic. The susceptibility of paramagnetic materials obeys the Curie Law,

$$\chi_m = \frac{\mu_0 C}{T} \tag{2}$$

Paramagnetic materials are attracted to magnets.

C. Diamagnets, linear materials with $-1 < \chi_m < 0$

If elementary particles did not have an intrinsic magnetic moment, then all materials would be diamagnetic. That is, the magnetic moment of materials would be opposite the direction of the applied field. This is due to Lenz's law. Superconductors are the best diamagnets, but many pure normal conductors are too (e.g. Cu...). At low enough values, magnetic fields are completely excluded from the interior of a superconductors. The phase in which this occurs is called the Meissner phase of a superconductor. From the expression,

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} \tag{3}$$

it is evident that in order for flux to be completely expelled so that $\vec{B} = 0$ inside the superconductor, we must have, $\chi_m = -1$. A measurement of χ_m is one of the first measurements that people do to determine if a material is in the superconducting state. *Diamagnetic materials are repelled from magnets.* This enables the possibility of magnetic levitation. Since superconductors are the best diamagnets, they are the primary candidates for possible magnetic levitation applications.

D. Ferromagnets, non-linear magnetic materials, hysteresis

In ferromagnetic materials, the magnetic moments of the atoms in the material seek to align in the same direction. Examples are Fe and Permalloy (55% Fe, 45% Ni). It is actually quite difficult to find good ferromagnetic materials. There is a continuing search for ferromagnetic materials which have large local magnetic moments. A group at GM research in Detroit made a major breakthrough in this area about a decade ago. They helped develop the Niodymium, Iron, Boron magnets. The production of these magnets is now a multibillion dollar industry. Calculation of the fields around magnetics is carried out in a similar manner to the fixed magnetization case discussed above, e.g. for a uniformly magnetized sphere. A more general calculation uses a non-linear consitutive law. Sometimes ferromagnets are treated as a linear dielectric with a large positive value of χ_m - this is not completely correct, but it gives an indication of the expected behavior.

Ferromagnetic materials are very important in technology. For example the hard drives in most computers are made using small domains on ferromagnetic materials. A small sensor (or read head) scans the surface of the hard drive. On the hard drive surface are small domains of ferromagnetic material. These domains are oriented in the plane of the surface and they have a prefered direction. The read head measures a resistivity which is sensitive to the local magnetic field. The technology of magnetic storage (e.g. hard drives) relies on a particular property of ferromagnetic materials. This property is called hysteresis. Hysteresis is a property which occurs when a magnetic field is applied to a ferromagnet which is below its Curie temperature.

In order to describe hysteresis we must describe the way in which we vary the temperature and the magnetic field. Let us start at high temperatures and quench to a temperature well below the Curie temperature. The magnetic material is frozen in a domain structure by this process. Now we apply a positive external field. The domains now begin to align with the magnetic field. At sufficiently high magnetic field the atomic magnetic moments are all aligned with the applied field. This is called the saturation magnetization.

Now consider reducing the applied field until it is oriented in the opposite direction to the direction of the magnetic moments. However, the magnetic moments in a ferromagnetic material prefer to have the same orientation so they do not want to follow the direction of the magnetic field at first. The magnetic moment then remains oriented opposite the applied field until a sufficiently large opposite magnetic field is applied. At this point a sudden switching of the orientation of the magnetic moment occurs. This is the switching field H_c. In magnetic storage, when we write information onto the hard drive, we are switching the orientation of the magnetic domains. The read operation does not do this, instead it just senses the direction of the local field. This magnetic memory is non-volatile as it is not necessary to have a power source continually applied to the material in order to maintain the orientation of the spins.

Magnetic materials with very large reversal fields (H_c)are called magnetically hard materials, while those with small hysteresis loops are called soft magnetic materials.

Antiferromagnetics and complex magnets, non-linear materials

Antiferromagnetic materials have atomic magnetic moments which prefer to have their nearest neighbors in an antiparallel alignment. In the simplest case, the magnetic moments alternate between one orientation and another. This is easily possible in material structures which are bipartite (e.g the square lattice or the cubic lattice), however in other lattice structures, the magnetic order can be extremely complex. These complex ordered states are called spin glasses or frustrated magnets. Antiferromagnetic materials loose their order at a critical temperature called the Neel temperature. Complex magnets loose their order at a glass temperature or ordering temperature. Often complex materials exhibit hysteresis and time-dependent effects that make reproducible measurements more difficult. There are many antiferromagnets and complex magnets. This is the usual behavior of compounds and some elements. Examples are NiO, Cr,...

E. Calculations involving linear magnetic materials

Example: Magnetic field enhancement in a solenoid containing iron

For a solenoid with n turns per unit length and carrying current I, we found,

$$B_0 = \mu_0 n I \tag{4}$$

This is the result for a solenoid in air. Now if we place a material inside the solenoid, we use Ampere's law for the field intensity, and $\vec{B} = \mu\vec{H}$, to find,

$$H = ni; \quad \text{and} \quad B = \mu H = \mu_0(1 + \chi_m)ni \tag{5}$$

From this expression it is evident that the magnetic field inside the solenoid is greatly enhanced if the center (the core) of the solenoid is composed of a magnetic material which has large magnetic susceptibility χ_m, for example permalloy. Note that this seems different to dielectrics where the electric field is reduced when a dielectric is placed between the plates of an isolated capacitor. However, if a capacitor is connected to a battery, the electric field is unaltered by the addition of the dielectric, however the charge stored increases by a large amount. If we associate the charge stored on the capacitor with the flux in the solenoid, then the two devices appear more similar. This is the analogy that is usually used.

The energy stored in the solenoid is still $Li^2/2$, but we need to calculated L again. L is defined through

$$Li = N\phi \quad \text{so that} \quad L = N^2\mu A/l \tag{6}$$

This is just the formula that we have for vacuum, but with $\mu_0 \to \mu$. The energy stored in the inductor thus increases dramatically when a large permeability material is used for the core of the inductor. As an example, consider an inductor containing permalloy with $\mu = 10000$ and with $N = 10,000$, $A = 0.1m^2$, $l = 1m$, carrying a current of $20A$, then $U = N^2\mu A i^2/l = 4 \times 10^13J$. This is a very large number and looks attractive for energy storage applications. However there are a number of critical limitations, ranging from hysteresis to resistive losses and the effects of large magnetic fields on materials and people.

Materials with positive χ_m are drawn toward regions of high field (i.e. the solenoid), while those with negative χ_m are repelled from regions of high field.

A co-axial cable

Consider a co-axial cable with an inner conductor of radius a and a material with permeability μ in the region $a < r < b$. A conducting cylinder completes the cable and has inner radius, b and outer radius c. Find the magnetic field in each of the four regions of the material. We did this problem before for the case of vacuum for the insulating material. The extension to this case is straightforward. Simply carry out the calculation using Ampere's law for $\vec{H}$ and then use $\vec{B} = \mu\vec{H}$ to find the field. Notice that we could do a problem where there are different values of μ in each regime in the same way, but just using the appropriate value of μ in each part of the co-axial.