PHY481 - Lecture 31

Section 10.3-10.5 of PS, Sections 7.2.3, 7.2.4 of Griffiths

Energy in EM waves

The classical EM energy density in vaccuum is composed of the electric and magnetic contributions,

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \epsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{c\mu_0} \tag{1}$$

where the latter two expressions are found by using $\vec{E} = c\vec{B}$ and $c = (\mu_0\epsilon_0)^{-1/2}$, and we only used the magnitudes of the fields. The energy flux density is the amount of energy crossing a surface per unit time. Lets consider a surface perpendicular to the direction of motion so that the amount of energy crossing a unit surface area is

$$Energy\ flux\ density = length * u/time = cu = \frac{EB}{\mu_0} \tag{2}$$

A compact way of writing this energy flux is to use the fact that the direction of propagation is the direction of $\vec{E}\wedge\vec{B}$, so it is natural to define the vector quantity that gives the magnitude and direction of the Energy flux density, the Poynting vector $\vec{S}$,

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \wedge \vec{B} \tag{3}$$

Now lets consider the quantum mechanical understanding of EM waves. In that case we know that each photon carries energy $h\nu$, so the energy flux density is the number of photons per unit time per unit area multiplied by $h\nu$,

$$Quantum\ energy\ flux\ density = n_\nu(t) h\nu = Intensity = Power/area \tag{4}$$

where $n_\nu(t)$ is the number of photons arriving at a surface per unit area per unit time. Note that this is the average energy flux density, whereas the Poynting vector gives the instantaneous power per unit area so the intensity in the classical case is $I = S_{av}$, where $S_{av} = E_0 B_0/(2\mu_0)$ is the time average of the Poynting vector and E_0, B_0 are the peak values of the electric and magnetic fields respectively.