Electroweak Corrections in High Energy Processes using Effective Field Theory

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Electroweak Sudakov corrections using effective field theory

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Electroweak Corrections using Effective Field Theory: Applications to the LHC

In Preparation

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Parton Processes

- Typical LHC processes being studied such as jet production, t-quark production, squark production proceed via energetic parton subprocess

$$qq \to qq, \qquad q\bar{q} \to q\bar{q}, \qquad q\bar{q} \to t\bar{t}, \qquad q\bar{q} \to \tilde{q}\tilde{q}^*$$

  with $Q \sim \sqrt{s}$ of order a few TeV.

- The final state invarient masses are much smaller than Q.

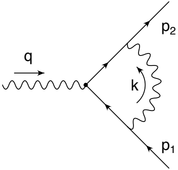
- Describe these states using an effective theory (SCET) and work in the regime

$$s \sim -t \sim -u \sim Q^2$$

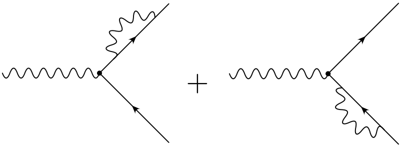
EM Form Factor: UV structure

A quick review

- The form factor for $J^{\mu}_{\text{EM}} = \overline{\psi}\gamma^{\mu}\psi$ diverges in both the UV and the IR regions.



- UV divergences cancel against the self-energy graphs (true since J^{μ}_{EM} is a conserved current)

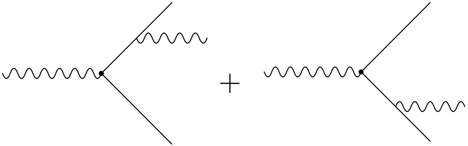


Form Factor: IR structure

- At 1-loop with external leg corrections using a fictitious boson mass (M_γ) gives:

$$F_E(Q^2) = \frac{\alpha}{4\pi}\left(-\log^2\frac{Q^2}{M_\gamma^2} + 3\log\frac{Q^2}{M_\gamma^2} - \frac{7}{2} - \frac{\pi^2}{3}\right)$$

- $\log\frac{Q^2}{M_\gamma^2}$ behaves badly as $M_\gamma \to 0$ (IR divergent)
- We are rescued by the “soft” bremstralung which is “undetectable” for $E_\gamma < E_{th}$.



- The offending IR divergences cancel, i.e. “IR safe”.

Electroweak Corrections

- What is different between QED and Weak interactions?
  - $W^{\pm}$'s, Z's are massive and are detectable in final state
  - $M \not\to 0$ so $F_E(Q^2)$ is IR finite
- expansion in $\frac{\alpha}{4\pi}$ is valid so long as $\log^2\left(\frac{Q^2}{M^2}\right)$ is small
- At LHC, a typical parton-parton interaction will exchange $\sqrt{s} \sim 1$ TeV ($q\bar{q} \to q\bar{q}$).

$$4 \times \frac{\alpha}{4\pi sin^2\theta_W} \log^2\left(\frac{Q^2}{M^2}\right) \sim 0.2$$

- Invalidates fixed order PT, must re-sum large logarithms

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General perturbative structure of $F_E(Q)$

$$Q \gg M \gg m_i \qquad L = \log \frac{Q^2}{M^2}$$

$$\begin{aligned}
F_E(Q) = \Bigg[1 \; &+ \left(\frac{\alpha}{4\pi s_W^2}\right)^1 \left(\mathrm{L}^2 + \mathrm{L}^1 + \mathrm{L}^0\right) && \text{LO + NLO} \\
&+ \left(\frac{\alpha}{4\pi s_W^2}\right)^2 \left(\mathrm{L}^4 + \mathrm{L}^3 + \mathrm{L}^2 + \mathrm{L}^1 + \mathrm{L}^0\right) && \text{N}^2\text{LO} \\
&+ \left(\frac{\alpha}{4\pi s_W^2}\right)^3 \left(\mathrm{L}^6 + \mathrm{L}^5 + \mathrm{L}^4 + \mathrm{L}^3 + \mathrm{L}^2 + \mathrm{L}^1 + \mathrm{L}^0\right)\Bigg] && \text{N}^3\text{LO} \\
&+ \left(\frac{\alpha}{4\pi s_W^2}\right)^4 \left(\mathrm{L}^8 + \mathrm{L}^7 + \mathrm{L}^6 + \mathrm{L}^5 + \cdots\right)\Bigg] && \text{N}^4\text{LO}
\end{aligned}$$

LL NLL N^2LL $\cdots$

EFT summary and strategy

- In QED it is well known that the leading logarithms (LL) exponentiate.

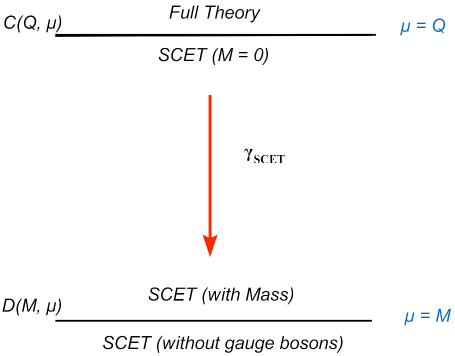
$$F_E(Q^2) = \exp\left[-\frac{\alpha}{4\pi}\log^2\left(\frac{Q^2}{M^2}\right)\right] \qquad \text{"Sudakov (1956)"}$$

Goal: To perform high energy calculations with a massive gauge boson and re-sum the logarithms.

1. Matching onto SCET at Q^2
2. run with the effective theory anomalous dim, γ_{SCET}, to a low energy
3. integrate out massive particles each time a mass threshold is passed (new EFT)

We will then have a "renormalization group improved" perturbation theory calculation with no large logarithms to spoil the expansion.

SU(2) “toy” theory with massive boson(M)

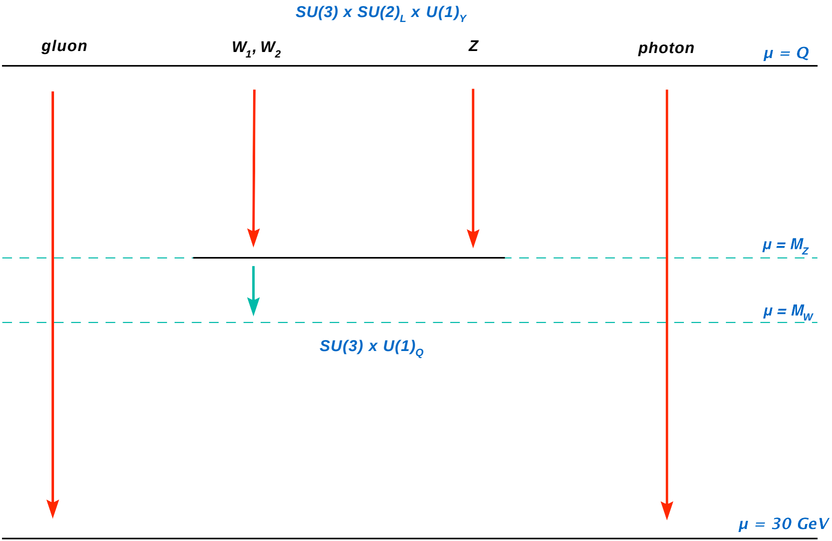


$$F_E(Q) = \exp\left[C(Q) + \int_Q^M \frac{d\mu}{\mu} \gamma_{\text{SCET}}(\mu) + D(Q, M) \right] \qquad (1)$$

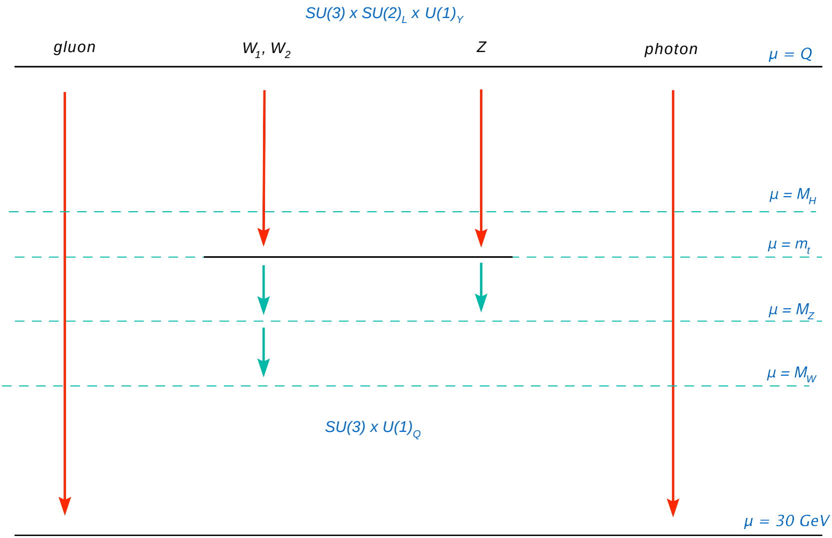
Standard Model

- Generalization to the standard model ($SU(3) \times SU(2)_L \times U(1)_Y$) is straight forward.
- We can treat unequal masses ($M_Z \neq M_W \neq M_H$). This is not clear using QCD inspired methods.
- The transition to $SU(3) \times U(1)_Q$ is straight forward.
  - Integrate out the W's and Z bosons simultaneously at $\mu = M_Z$.
  - No complications of an intermediate theory with W's and no Z's.
- Can match onto HQET to account for heavy particles (top quarks and SUSY particles)

light quarks

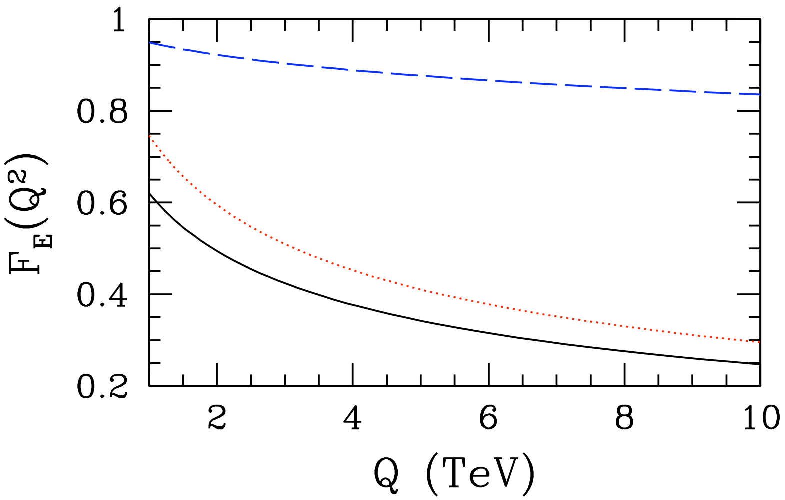


Top Quark



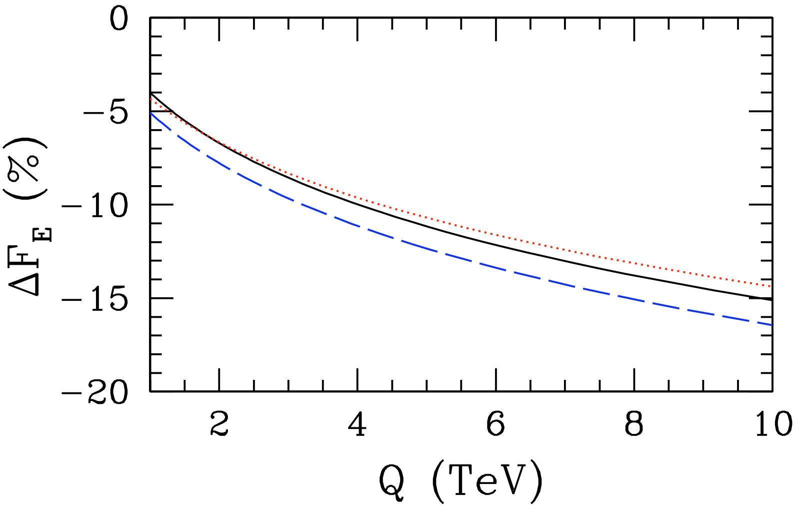
SU(3) x SU(2)_L x U(1)_Y
gluon
W_1, W_2
Z
photon
μ = Q
μ = M_H
μ = m_t
μ = M_Z
μ = M_W
SU(3) x U(1)_Q
μ = 30 GeV

$F_E(Q^2)$ for $M_h = 200 GeV$ and $\mu = 30 Gev$



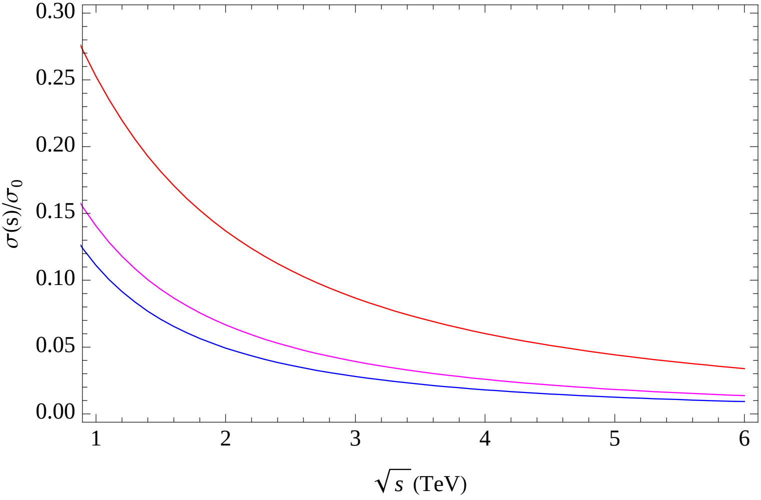
top quark   electron   up quark

$\Delta F_E(Q^2)$ due to EW corrections only



top quark electron up quark

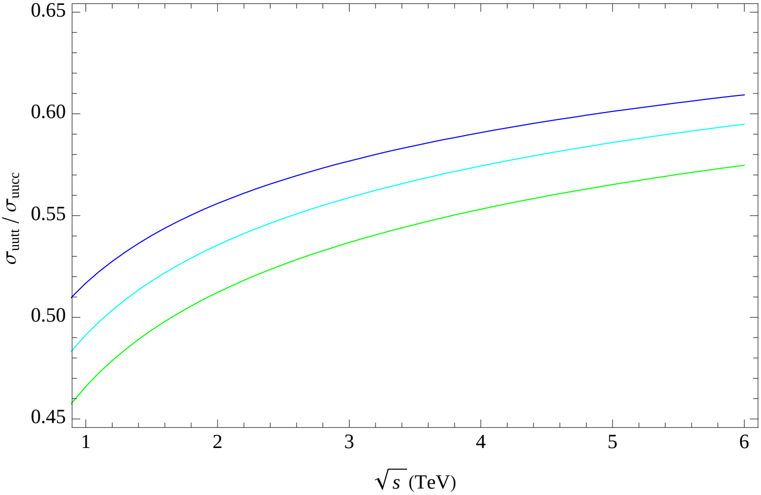
$u_L \bar{u}_L \to t_L \bar{t}_L$



$t = -0.1s$ $t = -0.3s$ $t = -0.5s$

Heavy Quark Mass Effects

$$\frac{\sigma_{uutt}}{\sigma_{uucc}}$$



$t = -0.5s$ $\quad t = -0.3s$ $\quad t = -0.1s$

Summary

- We have demonstrated the use of EFTs to sum the electroweak logarithms in a systematic way.
- Many processes can be considered in SM and SUSY.
- Mass effects due to differences in M_Z, M_W, M_h and m_t can be included.
- Outlook: the results can be easily generalized to include
  - gauge bosons:

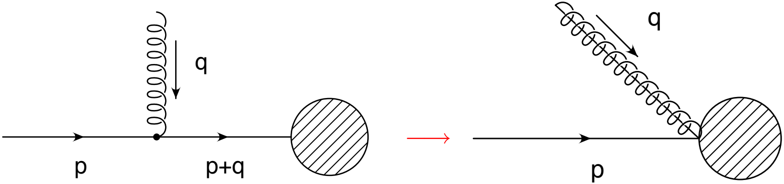
$$gg \to q\bar{q} \qquad\qquad q\bar{q} \to ZZ, WW, gg, \gamma\gamma, \cdots$$

  - Any number of widely separated jets.

Soft Collinear Effective Theory (SCET)

(Bauer, Fleming, Pirjol, Stewart)

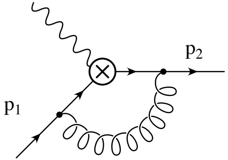
- Goal: Describe interactions between energetic particles ($E \sim Q$).
- A gauge boson with a large momentum can knock an on-shell external particle off its mass shell ($p^2 = m^2$)

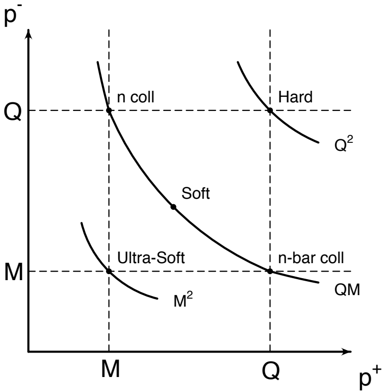


- Integrate out "far offshell" degrees of freedom.
- Neglects terms on the order of $\sim O(\frac{M^2}{Q^2})$ or $\sim O(\frac{m_i^2}{Q^2})$

SCET degrees of freedom (modes)

- full theory:





- EFT: